Abstract

Irrigation schedules are used in many irrigation schemes that operate on a rotational or arranged-demand basis. Determining these schedules is a complex problem, especially when done by hand. Operations research tools such as single machine scheduling are already used to schedule irrigation turns in systems where only one single user can irrigate simultaneously. This paper shows how multi-machine scheduling can be used to determine arranged-demand schedules for systems where two or more users can irrigate at the same time. Two models are presented; the simple multi-machine scheduling model that establishes a schedule for systems where all outlets/user discharges are identical and the complex multi-machine
scheduling model that determines the schedule when flows to individual outlet/users are not necessarily identical.

Keywords: water allocation; irrigation; machine scheduling; linear programming; complex machines.

Background

An irrigation agency, water user’s association or irrigation scheme’s board of management is no different from any other organization in that scarce resources, such as water and time, have to be allocated, in this case to water users, taking into account a large number of constraints. The water users in turn have their own demands, such as a preferred time to start irrigation or a certain amount of irrigation water requested. Matching these demands within the (physical) constraints of the irrigation systems can be a very complex task, especially when the number of users can reach dozens or even hundreds. The two common solutions to this problem are rotational irrigation and on-demand irrigation.

At one end of the spectrum in delivering a service – in this case irrigation water – is rotational irrigation. Rotational irrigation consists of granting each user (or group of users) exclusive use of all water in a channel for a certain length of time after which water is turned over in a sequence to the next user which is usually located directly
up or downstream from the current user. After each user has had their turn the cycle is repeated again and again (Zardari & Cordery, 2010). These cycles, or irrigation intervals as they are more commonly known, typically last 6 to 10 days. The benefits of this type of scheduling is that the process is straightforward, requires a relatively low investment in infrastructure and the skill level both of user and operations managers is low. Schedules can be prepared in advance, which can reduce conflict and uncertainty (Bhadra et al., 2010). However, this also reduces the flexibility of the system. Schedules are often prepared well before the actual irrigation takes place and no deviation from the planned rotation is allowed. With some schedules having been in place for years or possibly even decades, there is very little opportunity to change a schedule to account for changed circumstances such as different cropping patterns, drought or increased water supplies (Ghumman et al., 2009). As a result of this inflexibility rotational schedules can have low efficiencies which can have consequences such as yield reduction, high ground water tables and salinity (Qureshi et al., 2002).

At the other end of the spectrum is on-demand irrigation. In on-demand services such as electricity or telephone, the provided service is available instantly, in any quantity (subject to the limits of the systems) and for whatever duration the user requires the service. Similarly in an on-demand irrigation system water is available whenever required, in whatever quantity needed and for whichever flow rate desired, although some limits may apply to each of those and some advance notice (24-48 hours) may be required (Salvador et al., 2011). This allows water to be used when
and where most needed, which generally results in higher efficiencies and a reduction of the environmental and energy costs (Moreno et al., 2010). However, sufficient system capacity is needed to be able to meet all demands in a system, as in theory all users could request water at the same time. As a result such irrigation systems may have to include major storage reservoirs or pumping facilities. As flow rates can differ from user to user or over time, flow regulatory structures may be required. Operation of this type of system requires a high skill level as does determining water requirements. To be truly on-demand an irrigation system needs to have over-capacity, but this may be used only on a few occasions, if at all. As a result, the initial investment cost of an on-demand irrigation system can be very large and on-going operation & maintenance costs can be considerable.

Considering the large financial costs associated with on-demand irrigation and the large environmental costs associated with rotational irrigation, a possible solution is arranged-demand irrigation. In these systems there is sufficient capacity to allow a certain degree of flexibility, without being expensive in terms of the physical capacity required to deliver water. In times of peak demand the system may need to revert back to a simple rotational schedule, but any other time there would be enough capacity to allow some degree of on-demand scheduling. Arranged-demand scheduling requires a new schedule to be produce for each irrigation interval. Determining these schedules lies within the realm of operations research. This paper describes a method that uses operations research techniques to determine the most optimal order of irrigation once allocations are set. It does not determine the amount
of irrigation required, which can be calculated in many different ways, including based on crop-water requirements or pro-rata based on acreage.

For the determination of the irrigation amount, there can be two schools of thought – the first is to consider irrigation water as a service to be delivered at the time and amount as requested by the user. This is akin to any other service delivery e.g. telephone/electricity/potable water/internet where a service provider does not get involved in how efficiently the resource is used or to what purpose it is put. In the case of irrigation water it would be in everyone’s interest that the water resource is used as efficiently as possible and providing the user with information/tools to take an optimum decision when to use the resource. However in this school of thought it is for the end user to decide how and when to use the resource – ideally a well informed decision. The second school of thought is where an irrigation manager ‘knows best’ and provides the service when the manager thinks it is appropriate. The disadvantage of this approach is that it is likely the irrigation manager will only take crop requirements into consideration whereas an end user may factor in social and personal circumstances when making a decision. Factors that an irrigation manager is not privy to and nor can be included simply due to the number of end users involved and the complexity of human decision making. The authors are of the first school of thought.
As described by De Vries and Anwar (2004), allocating water to users in an irrigation system can be seen as a single machine scheduling problem. There is a single activity (giving water) and there are a number of events (water users) to be serviced. An irrigation scheme that operates under a rotational schedule, i.e. each user of a tertiary canal is granted the exclusive use of the water for a certain period, after which the water is turned over to the next user, is equivalent to the single-machine scheduling problem in Operations Research described by Baker (1974).

In single machine scheduling water is seen as a machine that can be used by at most one user at a time. This makes it a suitable analogy for rotational irrigation where users use the water sequentially. There are however many irrigation systems where two or more users irrigate simultaneously. Wang et al. (1995) introduced the concept of stream tubes. A stream tube is an imaginary sub-channel that supplies water to only one user at a time. After this user has finished the stream tube can supply the next user in the sequence. As water can divided into more than one of these imaginary stream tubes, more than one user can be supplied at a time. There is a clear analogy between the concept of stream tubes and machine scheduling. Each stream tube can be seen as a separate machine and multi-machine scheduling can be used to find the arranged-demand schedule for a number of users.
The work by Wang et al. (1995) focused on the decision whether to operate the outlets of a tertiary unit simultaneously or in sequence. Figure 1a shows how irrigating simultaneously requires a larger canal capacity (economically undesirable) and the availability of sufficient water resources. The large flow required to allow simultaneous irrigation can exceed the capacity of the channel. Figure 1b shows how operating the outlets sequentially allows the use of a smaller channel, but the total running time of a channel may exceed the irrigation interval. Figure 1c shows the optimal schedule, wherein all the outlets are supplied with water within the irrigation interval, using the smallest possible discharge.

Anwar and Clarke (2001) further developed the work by Wang et al. (1995) by showing how the concept of stream tubes can be used to schedule a number of events according to their target start times. A limitation of the work by both Wang et al. (1995) and Anwar and Clarke (2001) is that the discharges of all outlets are assumed to be identical.

Simple and complex multi-machine scheduling

Unlike real machines water can be divided. As shown above, if there is sufficient water in a canal it may be possible for two or even more users to use water simultaneously. In that case multi-machine scheduling allows water to be modeled as several imaginary stream tubes. Each machine or stream tube represents an identical unit of discharge. However, this allows very little flexibility. In many irrigation
systems it is possible for users not only to request the duration and start time for
their irrigation turn, but also to request a specific discharge. This means that the
discharge varies from outlet to outlet. By allowing more than one stream tube to
service an irrigation event it is possible to extend multi-machine scheduling to this
engineering application. This idea of allowing any number of machines or stream
tubes to service the same event at the same time is unique. There is no equivalent to
this concept in any other branch of machine scheduling. In those fields of study,
machines tend to be “real” physical machines that are limited in number. In irrigation
scheduling the number of machines is virtually unlimited, giving a huge amount of
flexibility not found anywhere else.

Figure 2 shows the conceptual difference between two types of multi-machine
scheduling. The case where all outlets receive the same discharge will be referred to
as simple multi-machine scheduling. The case where the discharges vary from outlet
to outlet and more than one stream tube may be needed to service that event will be
referred to as complex multi-machine scheduling. In simple multi-machine
scheduling each event is serviced by only one stream tube and each stream tube
can service several events in sequence as shown in Figure 2a. Three stream tubes,
A, B and C, each of equal discharge, are available. Stream tube A supplies user 1,
stream tube B supplies user 2 and stream tube C supplies user 3. All outlets receive
the same discharge. The limitation of such a model is that all outlets must have an
identical discharge. In complex multi-machine scheduling two or even more stream
tubes are allowed to service the same event, thereby permitting non-
identical/different discharges delivered to each outlet. Figure 2b shows this. Again, three stream tubes, A, B and C, each of equal size are available. Stream tube A supplies user 1, and stream tubes B and C supply user 2. Outlet 2 receives twice the discharge of user 1.

Mathematical formulation

The complex multi-machine model allows arranged-demand scheduling, i.e. scheduling of a number of events during an irrigation interval according to their target (requested) start times. In this model more than one stream tube is available to service each of the events, i.e. users are allowed to irrigate simultaneously. Events can be given priority by including a penalty or earliness/tardiness cost that is applied when an event is scheduled earlier or later than the target. For a detailed explanation on how to determine the penalty costs, see De Vries and Anwar (2004). A fictitious event with index 0 is used to simplify writing of the constraints. Let $Q = \{0,1,2,\ldots,N\}$ be a set of irrigation events to be scheduled. The following parameters are specified for each event $i \in Q$: duration, target start time, earliness cost, tardiness cost and required discharge. Also let $V = \{1,2,\ldots,W\}$ be a set of stream tubes available to service events. The discharge for each stream tube $w \in V$ is specified. In this model there are two decisions to be made, firstly which stream tube services which event and secondly what is the scheduled start time of each event? If the answers to these questions are known, the arranged-demand schedule is determined.
The objective of the model is to find an arranged-demand schedule such that every event starts as close as possible to the target start time. The objective function consists of two terms. The first aims to find the sequence of events and scheduled start time for each event so that every event starts as close as possible to the target start time. This is achieved by minimizing the penalties incurred when an event is either early or tardy. In irrigation scheduling the earliness and tardiness costs depend on the effect an early or late delivery of water can have on the irrigated crop. Both can have equally disastrous effects and are often taken to be the same. In that case, the earliness and tardiness costs can be set to 1. However Anwar and Clarke (2001) and De Vries and Anwar (2004) describe how earliness/tardiness costs can be used to improve multi-interval scheduling decisions.

The second term aims to limit the wastage of water by reducing the discharge in the channel. This can be done by minimizing the number of stream tubes used. Multiplying the second term by a large constant will ensure that minimizing the discharge has priority over the minimizing the earliness/tardiness. This constant will have to be chosen such that the second goal will be several orders of magnitude larger than the first, thus ensuring the priority of the second goal. Although the value of the objective function will be influenced by the chosen value of the constant, it does not influence the number of stream tubes and scheduled start times for the outlets (Anwar and Clarke, 2001). The objective function can be written as
minimize $\sum_{i=1}^{N} (\alpha_i E_i + \beta_i T_i) + c \sum_{w=1}^{W} \psi_w$ \hspace{1cm} (1)

where: $\alpha_i =$ cost of earliness per unit of time for event $i$; $E_i =$earliness of event $i$; $\beta_i =$ cost of tardiness per unit of time for event $i$; $T_i =$tardiness of event $i$; $c =$ large constant; $\psi_w =$ binary variable which defines whether stream tube $w$ is activated; $N =$ number of events to be scheduled; $i,j =$ index representing event $(0),1,2,\ldots,N$; $w =$ index representing stream tube $1,2,\ldots,W$; and $W =$ number of available stream tubes.

**Decision variables**

Any event can directly precede any other event on a stream tube; therefore, a variable is used to define which event directly precedes which other event on what stream tube.

\[ \phi_{ijw} = \begin{cases} 1 & \text{if job } i \text{ directly precedes job } j \text{ on stream tube } w \\ 0 & \text{otherwise} \end{cases} \quad i \neq j \] \hspace{1cm} (2)

where: $\phi_{ijw} =$ binary variable which defines if event $i$ directly precedes event $j$ on stream tube $w$.  


An event can be serviced by any stream tube, so a variable is used to define which event is serviced by which stream tube.

\[ \tau_{iw} = \begin{cases} 1 & \text{if job } i \text{ is processed by stream tube } w \\ 0 & \text{otherwise} \end{cases} \]  

(3)

where: \( \tau_{iw} \) = binary variable which defines if event \( i \) is serviced by stream tube \( w \)

The following variable defines whether a stream tube is used to service any event (activated stream tube) or otherwise

\[ \psi_w = \begin{cases} 1 & \text{if stream tube } w \text{ is activated} \\ 0 & \text{otherwise} \end{cases} \]  

(4)

**Constraints**

The scheduled start time of an event is determined from the target start time, the earliness, and the tardiness

\[ S_i = r_i + T_i - E_i \quad \forall i \]  

(5)

where: \( S_i = \) actual start time of event \( i \); and \( r_i = \) target start time of event \( i \).
Earliness and tardiness are mutually exclusive and therefore cannot take negative values.

\[ T_i \geq 0 \quad \forall i \quad \text{and} \quad E_i \geq 0 \quad \forall i \]  (6)

Each outlet must receive the required discharge; therefore, each event must be serviced by the right number of stream tubes.

\[ q \sum_{w=1}^{W} \tau_{iw} = Q_i \quad \forall i \]  (7)

where: \( q \) = discharge represented by one stream tube; and \( Q_i \) = discharge required for event \( i \).

Each event can at most precede one other event.

\[ \sum_{j=1}^{N} \phi_{ij} \leq \tau_{iw} \quad \forall i; \forall w; i \neq j \]  (8)

Each event must follow one other event. Using this constraint without introducing the fictitious event indexed 0 is impossible for the very first event in the schedule. Using
event indexed 0, which has no duration, makes it possible to create an exception to this constraint without affecting the real events.

\[
\sum_{i=0}^{N} \phi_{ijw} = \tau_{jw} \quad \forall j; \forall w; i \neq j
\]  

No event is allowed to start before the previous event on the stream tube has finished

\[
S_j - S_i + M(1 - \phi_{ijw}) \geq d_i \quad \forall i; \forall j; \forall w; i \neq j
\]  

where: \( M = \) a large positive number; and \( d_i = \) duration of event \( i \). The inclusion of \( M \) ensures that equation (10) behaves as intended i.e. equation 10 becomes redundant when the variable \( \phi_{ijw} = 0 \). The solution is not sensitive the value of the positive constant \( M \), provided that it is sufficiently large – orders of magnitude greater than the irrigation interval.

Each event should be finished in the interval

\[
S_i + d_i \leq g \quad \forall i
\]  

where: \( g = \) length of irrigation interval.
A stream tube is activated if it services at least one event (supplies at least one outlet with water).

\[ \sum_{i=1}^{N} \tau_{iw} \leq \psi_{w}N \quad \forall w \]  

The capacity of a channel must not be exceeded

\[ q \sum_{w=1}^{W} \psi_{w} \leq Q_{c} \]  

where: \( Q_{c} \) = total discharge in a channel.

The complex multi-machine scheduling problem model is therefore defined by the objective function (1), the decision variables (2)-(4) and the constraints (5) - (13).

**Helpful constraint**

Although not strictly necessarily including the following constraint will speed up solving the model by determining the minimum number of stream tubes required to fit
every event into the irrigation interval: for each stream tube the sum of the durations must be less than or equal to the length of the interval

\[ \sum_{i=1}^{N} d_i \tau_{iw} \leq g \quad \forall w \tag{14} \]

Selection of \( q \)

The selection of the discharge represented by a single stream tube is somewhat arbitrary and can simply be selected as 1 L/s if all outlet discharges are integer values in litres per second. However in order to reduce the size of the problem to be solved the discharge of each stream tube should be represented by the greatest common divisor of all outlet discharges.

Timeliness

One of the most important activities of irrigation district managers is the timely delivery of water to tertiary units, the smallest sub division within an irrigation scheme (Javan et al., 2002). Some irrigation districts have rules and policies regarding the late or early delivery of water, e.g. water must be delivered within 24 or 48 hours of the requested time (e.g. Palmer et al, 1991; McCornick, 1993).

Discharges in the channels can be increased, if the capacity of the channels and the availability of water permit this. This will allow more users to irrigate simultaneously. This increases the flexibility of an irrigation schedule and can reduce the
earliness/tardiness. If the discharge can be increased sufficiently it is possible to give
every user water exactly on time, this makes the system effectively an on-demand
service. This could however lead to excessive wastage of water, an excessive
number of gate operations, and increased cost of pumping (and use of energy) and it
may require channel sizes to increase beyond what is economically and/or physically
feasible. The necessary increase in discharge depends on the requested start times
and durations and can therefore only be determined together with the schedule. By
combining a model that minimizes the discharge and a model that determines the
earliness and tardiness, it is possible to develop the model that allows both
restrictions on earliness/tardiness and minimizes operational spillage at the same
time.

Anwar et al. (2006) also suggested scheduling models should limit the maximum
earliness/tardiness and defined a dimensionless number between 0 and 1 as relative
timeliness. A value of 1 indicated an on-demand schedule where there is no
earliness/tardiness. A value of relative timeliness of 0 indicates a schedule where
there is no restriction on earliness/tardiness other than an event must be scheduled
and completed within the interval.

Restrictions on maximum allowable earliness and tardiness can be formulated as
follows
\[ E_i \leq E_{\text{max}} \quad \forall i = 1, 2, \ldots, N \]  

(15)

where \( E_{\text{max}} \) = maximum allowable earliness, and

\[ T_i \leq T_{\text{max}} \quad \forall i = 1, 2, \ldots, N \]  

(16)

where \( T_{\text{max}} \) = maximum allowable tardiness. Alternatively using relative timeliness, restrictions on earliness/tardiness can be imposed as follows

\[ E_i \leq (1 - R_i) g \quad \forall i = 1, 2, \ldots, N \]  

(17)

and

\[ T_i \leq (1 - R_i) g \quad \forall i = 1, 2, \ldots, N \]  

(18)

where \( R_i \) = relative timeliness

Two-stage formulation

The model described in the previous paragraphs has a dual goal objective function: not only is the earliness/tardiness is minimized, but also the number of stream tubes needed to service all events. It is possible to separate these two goals into a two stage model, the first stage of the model minimizes the number of stream tubes
needed and the second stage of the model minimizes the earliness/tardiness. The
advantage of a two stage model rather than the single-stage model described earlier
is that the number of variables and constraints is reduced. As a result the model is
computationally less demanding and can be solved faster. However, the two stage
model does not allow simultaneous determination of the number of stream tubes and
earliness/tardiness. This is required when earliness/tardiness restrictions apply such
as the timeliness described before. Therefore the use of the two-stage model
described below is more restricted than the one-stage model, but can be used in
cases where timeliness is not an issue.

Stage 1 to minimize the number of stream tubes

The aim of the first stage is to minimize the discharge in the channel, i.e. to minimize
the number of activated stream tubes; therefore, the objective function reduces to

\[
\text{minimize} \sum_{w=1}^{W} \psi_w
\]

(19)

All constraints and decision variables regarding the actual scheduling of events can
be removed, as they will be applied at the second stage. The first stage model is
now defined by objective function (19), decision variables (3) and (4) and constraints
(7), (12) and (14).
Stage 2 to minimize earliness/tardiness

The second stage of the two stage model closely resembles the single-stage model described previously. As the objective of the model is no longer to both minimize the number of stream tubes and earliness/tardiness, but only to minimize earliness/tardiness, the second term of the objective function in (1) can be removed to give

\[
\text{minimize} \sum_{i=1}^{N} (\alpha_i E_i + \beta_i T_i) \quad (20)
\]

As the number of stream tubes that is needed is determined with the model described in a previous section, any constraints and decision variables in the previous model directly linked to this goal can be removed. The second stage of the model for non-contiguous complex multi-machine scheduling is now defined by the objective function (20), the decision variables (2) and (3) and the constraints (5) - (11) and (13). Constraint (14) can be used to reduce solution times.

Lower bound

Stage 1 of the two stage model described in the previous section minimizes the number of stream tubes required to service all events while ignoring any earliness/tardiness constraints. This can also be used to determine the lower bound for the single-stage model, which can result in reduced computation times.
Examples

The models for both simple and complex multi-machine scheduling are computationally very demanding and only relatively small problems can be solved. Although in the literature several tertiary units have been described that could be used for applying the models developed in this paper, all these tertiary units are too large to be solved in a reasonable computational time. Suryavanshi and Reddy (1986) described a tertiary unit in an irrigation scheme in India with 8 users who are allowed to irrigate simultaneously. In their paper a schematic representation and irrigation durations for this tertiary unit are given. Figure 3 shows this schematic and Table 1 shows the irrigation durations. Anwar and Clarke (2001) generated uniformly distributed random target start times for each event within the tertiary unit described by Suryavanshi and Reddy (1986). Table 1 also shows these target start times. This tertiary unit is used to show the differences between simple multi-machine scheduling, simple multi-machine scheduling with timeliness restrictions and complex multi-machine scheduling.

Bishop and Long (1984) describe a 16-user irrigation scheme in the Philippines. Randomly generated uniformly distributed target start times for this scheme can be found in De Vries and Anwar (2004). Both tertiary units mentioned here have been used to show the increased solution times with increased size of the tertiary unit.
Simple multi-machine scheduling

Figure 4 shows the arranged-demand schedule obtained with the complex machine scheduling model for the 8-user tertiary unit shown in Figure 3 and Table 1. Coincidentally the discharge of all outlets is identical, 30 L/s, and therefore could have been treated as a simple multi-machine scheduling problem. It can be seen that three stream tubes are needed to supply the outlets with water. The total earliness/tardiness for this schedule is 4.73 days, an average of 15.9 hours per outlet. This is the same result presented by Anwar and Clarke (2001). Figure 4 also shows that event 1 is scheduled to start 1.01 days later than requested and event 7 is scheduled to start 2.17 days later than requested, all other events have an earliness/tardiness of less than 1 day.

Simple multi-machine scheduling with timeliness constraints

The solution described in Figure 4 results in a delay of 2.17 days for one of the users. A delay this long can be detrimental to crop growth and in some irrigation systems a requirement of timely delivery of water has been introduced. Figure 5 shows the arranged-demand schedule if a maximum allowable earliness/tardiness of 24 hours per outlet is imposed on the example of the 8-user tertiary unit. The figure shows that four machines or stream tubes, each of 30 L/s, the greatest common divisor of all outlet discharges can accommodate this constraint. The total earliness/tardiness of this schedule is 1.44 days, an average of 4.3 hours per outlet. None of the outlets has an earliness/tardiness of more than 1 day.
The restrictions on maximum allowable earliness/tardiness do however have disadvantages. The discharge in the channel needs to increase from 90 l/s to 120 l/s. If the channel is operated continuously during the irrigation interval, 49% of the water is lost to operational spillage (volume of water lost as a proportion of that requested) compared to 12% operational water loss if no restrictions are imposed. This operational spillage shows as idle time in the schedule. Depending on the irrigation system, closing intakes and gates can prevent this operational spillage.

Complex multi-machine scheduling

The tertiary unit described in Figure 3 and Table 1 can be used as a basis to demonstrate the application of the complex multi-machine arranged-demand schedule. The required discharge of two randomly chosen outlets (outlet 2 and 5) are doubled and trebled to 60 L/s and 90 L/s respectively. If one stream tube delivers 30 L/s, this means that two stream tubes need to service outlet 2 and three stream tubes need to service outlet 5. The two-stage model described previously will be used to find the optimal schedule.

The first stage of the model determines that five stream tubes delivering 150 L/s in total are needed to supply the 8 outlets with the required discharge. Figure 6 shows the schedule obtained after applying the both the first and second stages of the
complex multi-machine model. It can be seen that three stream tubes (B, C and D) each deliver 30 L/s to outlet 5 giving 90 L/s as required. It can also be seen that two stream tubes (C and D) each deliver 30 L/s to outlet 2, giving a total of 60 L/s. Finally it can be seen that the remaining 6 outlets each receive 30 L/s delivered by one stream tube.

**Solution times**

The models describe in the previous sections were implemented in Lingo 13.0.2.16®. The complex multi-machine scheduling model and the alternative formulations were run with 8 users (using the data in Figure 3 and Table 1) and 16 users (using the data given in De Vries and Anwar, 2004). The results of the single-stage model, the two-stage model, and the single-stage model with a lower bound constraint are shown in Table 2. The single-stage model uses equation 1 to determine the schedule. Constant $c$ in this equation was set at 100. This means that the number of stream tubes is multiplied by 100. This does not change the actual schedule, but makes it easier to separate the required stream tubes from the incurred earliness/tardiness. For instance, table 2 shows a solution of 404.89 which implies 4 stream tubes are required ($4 \times 100 = 400$) and the sum of earliness/tardiness over all events is 4.89 days. If constant $c$ had been set at 1 rather than 100, the solution would have been $4 + 4.89 = 8.89$, making the solution less intuitive to interpret.
Table 2 shows that neither of the two single-stage models solved within the allocated computational time of 24 hours. Imposing a lower bound on the single-stage model does improve the solution time for the 8-event model significantly and a solution was obtained in just over 9 hours. Separating the model into two stages improves the solution time even further to a combined total of less than 5 minutes. None of the models for the larger 16-events model resulted in a solution within the allowed time. It can be seen that the objective value of the two-stage model is lower than those of the single-stage model and the single-stage model with an lower bound constraint. As this is a minimization model, a lower objective value is a better solution. It does however not necessarily mean that an optimal solution would have been found sooner with the two stage model than with either of the two other models if more time was available. It also does not necessarily mean that this objective value is anywhere near the optimal objective value.

Conclusions

An irrigation agency is no different from any other organization in that limited resources have to be allocated taking into account certain demands and constraints. There is however, one significant difference, whereas in other machine scheduling situations the machines are real and limited in number, water can be divided into any number of imaginary stream tubes. By allowing each event to be serviced by more than one stream tube at the same time, it is possible to deliver any discharge to an outlet. This unique feature enables scheduling of outlets that have varying discharges. The complex multi-machine model presented in this paper takes
advantage of this concept. The model can also take into account management policies on maximum allowable earliness and/or tardiness. The method described in this paper is a useful tool in arranged-demand irrigation systems.

The model however does have some shortcomings:

1. Water takes time to travel from one outlet to another. This time is completely dependent on the order in which irrigation takes place. Sequence dependent setup times should be included in the model used in those situations. The model as it currently stands is however useable in piped systems where travel times play no role.

2. Solution times increase with the number of events to be scheduled and as a result, only smaller problems can be solved. Solution techniques such as heuristics or genetic algorithms may be more appropriate to solve larger problems, in which case the algorithm presented herein can provide benchmark solutions for small problems.

Notation

c = large constant

d_i = duration of event i

E_i = earliness of event i

E_{\text{max}} = maximum allowable earliness
\( g = \) length of irrigation interval

\( i, j = \) index representing event \((0),1,2,\ldots,N\)

\( M = \) a large positive number

\( N = \) number of events to be scheduled

\( q = \) discharge represented by one stream tube

\( Q_c = \) total discharge in a channel

\( Q_i = \) discharge required for event \( i \)

\( R_i = \) relative timeliness

\( r_i = \) target start time of event \( i \)

\( S_i = \) actual start time of event \( i \)

\( T_i = \) tardiness of event \( i \)

\( T_{\text{max}} = \) maximum allowable tardiness

\( w = \) index representing stream tube \( 1,2,\ldots,W \)

\( W = \) number of available stream tubes

\( \alpha_i = \) cost of earliness per unit of time for event \( i \)

\( \beta_i = \) cost of tardiness per unit of time for event \( i \)
\( \tau_{iw} \) = binary variable which defines if event \( i \) is serviced by stream tube \( w \)

\( \varphi_{ijw} \) = binary variable which defines if event \( i \) directly precedes event \( j \) on stream tube \( w \)

\( \psi_w \) = binary variable which defines whether stream tube \( w \) is activated

References


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Figure 5 Simple multi-machine scheduling problem with timeliness restrictions (24 hours)
Figure 6 Complex multi-machine schedule
List of figures with matching captions (figures are also supplied as separate TIFF files)

Figure 1 Simultaneous and sequential irrigation
Figure 2 Simple and complex multi-machine scheduling

Figure 3 Tertiary unit Kukadi project, India with Outlet numbers, outlet size and distance from branch canal (after Suryavanshi and Reddy, 1986)
Figure 4 Irrigation schedule for simple complex machine scheduling problem

Figure 5 Simple multi-machine scheduling problem with timeliness restrictions (24 hours)
Figure 6 Complex multi-machine schedule
Table 1 Outlet data (Suryavanshi and Reddy, 1986; Anwar and Clarke, 2001)

<table>
<thead>
<tr>
<th>Outlet number</th>
<th>Irrigation duration (days)</th>
<th>Target start time (days)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>0.80</td>
<td>3.55</td>
</tr>
<tr>
<td>2</td>
<td>2.13</td>
<td>0.41</td>
</tr>
<tr>
<td>3</td>
<td>2.40</td>
<td>2.16</td>
</tr>
<tr>
<td>4</td>
<td>1.72</td>
<td>1.49</td>
</tr>
<tr>
<td>5</td>
<td>2.05</td>
<td>0.61</td>
</tr>
<tr>
<td>6</td>
<td>2.43</td>
<td>0.26</td>
</tr>
<tr>
<td>7</td>
<td>2.05</td>
<td>1.60</td>
</tr>
<tr>
<td>8</td>
<td>2.50</td>
<td>3.03</td>
</tr>
</tbody>
</table>
Table 2. Solution times for single stage, single stage with lower bound and two stage models

<table>
<thead>
<tr>
<th>Model</th>
<th>No of jobs</th>
<th>Length of interval (days)</th>
<th>Solution time (hh:mm:ss)</th>
<th>Objective value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Single</td>
<td>8</td>
<td>7</td>
<td>&gt;24:00:00*</td>
<td>404.89</td>
</tr>
<tr>
<td>Lbound</td>
<td>8</td>
<td>7</td>
<td>09:01:39</td>
<td>404.89</td>
</tr>
<tr>
<td>Stage 1</td>
<td>8</td>
<td>7</td>
<td>&lt;00:00:01</td>
<td>4</td>
</tr>
<tr>
<td>Stage 2</td>
<td>8</td>
<td>7</td>
<td>00:04:36</td>
<td>4.89</td>
</tr>
<tr>
<td>Single</td>
<td>16</td>
<td>7</td>
<td>&gt;24:00:00*</td>
<td>423.03</td>
</tr>
<tr>
<td>Lbound</td>
<td>16</td>
<td>7</td>
<td>&gt;24:00:00*</td>
<td>424.84</td>
</tr>
<tr>
<td>Stage 1</td>
<td>16</td>
<td>7</td>
<td>00:00:01</td>
<td>4</td>
</tr>
<tr>
<td>Stage 2</td>
<td>16</td>
<td>7</td>
<td>&gt;24:00:00*</td>
<td>14.7</td>
</tr>
</tbody>
</table>

* note that model runs were abandoned after 24 hours if no solution was obtained