Estimation of Soil Parameters of Coastal Areas by The Data of Well

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ABSTRACT

Taiwan is an island around with sea, there are many plowland near coastal areas, and soil parameters of the plowland must be known when calculating the water demand of irrigation, so estimation of soil parameters of coastal areas is a very important work. Estimation of soil parameters by the traditional method: the pumping test, which may have some errors due to the vibration of groundwater level influenced by morning and evening tides, which is a moving boundary condition different from the assumption of pumping test. This paper use the Bousniseq equation as the governing equation, and apply the variation of groundwater level between two wells, so that we can get the analytical solution of groundwater level between two wells, by using the other data of the two wells, the soil parameters in the Bousniseq equation can be estimated by the inverse method. In this paper, we apply the data of the coastal area near the Australia on April 1989 to make verification, the estimation of soil parameters are matched.

Key Words: Well, Beach, Groundwater

1.INTRODUCTION

The groundwater level near coastal areas vibrate due to the complicated variation of sea water level, the influence is inverse proportion to the distance between coast and well, and therefore estimation of soil parameters of coastal areas is restricted to boundary condition, assumption and pumping test. But Taiwan is an island around with sea, there are many plowland near coastal areas, and soil parameters of the plowland must be known when calculating the water demand of irrigation, so estimation of soil parameters of coastal areas is a very important work. The are two main influences of sea water level are tides and waves, tides influences a lot on the time and space scale, and waves influences on a little vibration of amplitude. Dominick et. al.(1971) neglected the influences of waves and assumed that the boundary of coast is a vertical wall. Nielsen (1990) considered the angle of inclination of seabed and made a comparison between theoretical value and observed 11 well data, he thought that the linearization of governing equation and neglecting of the seepage cause the error between theoretical value and observed data. Nielsen (1997) considered the vertical velocity caused by tides and waves, and derived a new governing equation. Li et. al. (2000) applied the governing equation derived by Nielsen (1997) and considered influences of unsaturated layer; he thought that unsaturated layer influences the water level of saturated layer. This paper assumed that the aquifer near coastal areas are isotropy and homogeneous, and with impermeable bottom boundary. We applied Boussinesq's equation as governing equation and periodic amplitude of groundwater level of two different wells as moving boundary conditions, we obtain the analytical solution of groundwater level, in addition, we apply the groundwater level of the other one well to estimate the soil parameters by inverse method, in this paper, we used data observed by Nielsen (1990).

2. THEORY

Let's consider an unconfined aquifer with two moving boundary conditions with periodic function of Sin, governing equations can be written as

$$
\begin{aligned}
&\frac{\partial}{\partial x}\left[h\frac{\partial h}{\partial x}\right]=\frac{n}{K}\frac{\partial h}{\partial t},\ l>x>0\\
&h(x,0)=f(x)\\
&h(0,t)=\phi_1(0,t)\\
&h(l,t)=\phi_2(l,t)
\end{aligned}
\tag{1}
$$

where h : groundwater level (L), n : porosity of soil(dimensionless) and K : permeability coefficient of soil(L/T).

The linearized governing equation can be written as

$$
D\frac{\partial}{\partial x}\left[\frac{\partial h}{\partial x}\right]=\frac{n}{K}\frac{\partial h}{\partial t},\ l>x>0 \quad \textbf{(Nielsen, 1990)}
\tag{2}
$$

where D : average water level of aquifer (L), and let $k=\frac{DK}{n}$, then the governing equation can be written as

$$
\begin{aligned}
&\frac{\partial^2 h}{\partial x^2}=\frac{1}{k}\frac{\partial h}{\partial t},\ l>x>0\\
&h(x,o)=f(x)\\
&h(0,t)=\phi_1(0,t)\\
&h(l,t)=\phi_2(l,t)
\end{aligned}
\tag{3}
$$

Equation (3) were solved by Carslaw & Jaeger (1959, P102~104) can be written as

$$h(x,t)=\tfrac{2}{L}\cdot\sum_{n=1}^{\infty} e^{-kn^2\pi^2 t/L^2}\cdot\sin\tfrac{n\pi x}{L}\cdot\left[\int_0^L f(x')\sin\tfrac{n\pi x'}{L}dx'+\tfrac{n\pi k}{L}\int_0^t e^{\frac{kn^2\pi^2\lambda}{L^2}}(\phi_1(\lambda)-(-1)^n\phi_2(\lambda))d\lambda\right] \tag{4}$$

Consider the initial condition and boundary equation as

$$\begin{aligned} f(x) &= a_0x^2+b_0x+c_0 \\ \phi_1(0,t) &= a_1+b_1\cdot\sin(2\pi\tfrac{t-d_1}{c_1}) \\ \phi_2(l,t) &= a_2+b_2\cdot\sin(2\pi\tfrac{t-d_2}{c_2}) \end{aligned} \tag{5}$$

Then Equation (4) can be written as

$$\begin{aligned} h = {} & \tfrac{2}{l}\sum_{n=1}^{\infty} e^{-k(\frac{n\pi}{l})^2 t}\cdot\sin\tfrac{n\pi x}{l}\{\tfrac{-2a_0}{(\frac{n\pi}{l})^3}+\tfrac{c_0}{(\frac{n\pi}{l})}+(-1)^n\cdot[\tfrac{-(\frac{n\pi}{l})^2 l^2 a_0+2a_0}{(\frac{n\pi}{l})^3}-\tfrac{lb_0+c_0}{(\frac{n\pi}{l})}] \\ & +\tfrac{n\pi}{l}k\{\tfrac{a_1}{k(\frac{n\pi}{l})^2}(e^{k(\frac{n\pi}{l})^2 t}-1)+\tfrac{b_1}{(k(\frac{n\pi}{l})^2)^2+(\frac{2\pi}{c_1})^2}\{e^{k(\frac{n\pi}{l})^2 t}\cdot[k(\tfrac{n\pi}{l})^2\sin(\tfrac{2\pi}{c_1}(t-d_1)) \\ & -\tfrac{2\pi}{c_1}\cos(\tfrac{2\pi}{c_1}(t-d_1))]-k(\tfrac{n\pi}{l})^2\cdot\sin(\tfrac{2\pi}{c_1}(-d_1))+\tfrac{2\pi}{c_1}\cos(\tfrac{2\pi}{c_1}(-d_1))\} \\ & -(-1)^n\{\tfrac{a_2}{k(\frac{n\pi}{l})^2}(e^{k(\frac{n\pi}{l})^2 t}-1)+\tfrac{b_2}{(k(\frac{n\pi}{l})^2)^2+(\frac{2\pi}{c_2})^2}\{e^{k(\frac{n\pi}{l})^2 t}\cdot[k(\tfrac{n\pi}{l})^2\sin(\tfrac{2\pi}{c_2}(t-d_2)) \\ & -\tfrac{2\pi}{c_2}\cos(\tfrac{2\pi}{c_2}(t-d_2))]-k(\tfrac{n\pi}{l})^2\cdot\sin(\tfrac{2\pi}{c_2}(-d_2))+\tfrac{2\pi}{c_2}\cos(\tfrac{2\pi}{c_2}(-d_2))\}\}\}\} \end{aligned} \tag{6}$$

3. EXPERIMENTAL DATA

Nielsen (1990) observed groundwater level of 11 wells at coastal areas near the south of Sydney in Australia, the distances between two wells is 2.5m and measured time was 1989/04/18-1989/04/19, he measured groundwater level every half an hour, the measured data of well No.7 to No.11 are shown in Table 1 and can be written as function of Sin as equation (7)-(11). The plot of measured data of well No.7 to No.11 and equation (7)-(11) are shown in Figure (1)-(5).

$$\phi_7(t) = 0.285 + 0.2 \cdot \sin(2\pi \tfrac{t-155}{745}) \tag{7}$$

$$\phi_8(t) = 0.285 + 0.14 \cdot \sin(2\pi \tfrac{t-135}{745}) \tag{8}$$

$$\phi_9(t) = 0.285 + 0.107 \cdot \sin(2\pi \tfrac{t-165}{745}) \tag{9}$$

$$\phi_{10}(t) = 0.285 + 0.087 \cdot \sin(2\pi \tfrac{t-172}{745}) \tag{10}$$

$$\phi_{11}(t) = 0.285 + 0.075 \cdot \sin(2\pi \tfrac{t-175}{745}) \tag{11}$$

From equation (7)-(11) we can find that the amplitude of groundwater level is inverse proportion to the distance between well and coast, the influences of tide become obvious for the well which close to coast.

4. MODEL CALCULATION

In this paper, we use Nielsen's (1990) data and simulate two cases, case 1 uses data of well No.7 and No.9 as moving boundary conditions to calculate the groundwater level of well No.8; case 2 uses data of well No.9 and No.11 as moving boundary conditions to calculate the groundwater level of well No.10, assume the initial time is 1989/04/18 PM 2:00, then the initial condition of case1 and case2 can be written in equation (12) and (13) as

$$f(x) = -0.0022x^2 + 0.0239x + 0.107 \tag{12}$$

$$f(x) = -0.0017x^2 + 0.0166x + 0.1717 \tag{13}$$

Substituting initial conditions and boundary conditions into equation (6) gives the groundwater level of well No.8 and No.10, which are close to the observed data by Nielsen's (1990), the comparison of analytical solution and observed data for case1 and case2 are shown in Figure (6) and (7). The permeability coefficient of soil can be calculated by the method of inversion, we obtain that k=2(m^2/min) for case1 and k=1.4(m^2/min) for case2.

5. RESULTS AND DISCUSSIONS

From Figure (6) and (7) we can find out that the analytical solutions are close to the observed data, but the permeability coefficient of soil k for case1 and case2 are different, for case1 well No.8 is more close to coast than well No.10, so value of k is bigger. Substituting value of k for case2 into case1 to calculate the groundwater level, we can find out that the simulated groundwater level are higher than the observed data, which is shown in Figure (8). The reasons for the difference between case1 and case2 are: (1) the well, which close to coast does have large k. (2) The governing equation Boussinesq's equation only consider the saturated zone, which neglects the influence of water content in unsaturated zone (Li et.al.2000), for the area close to coast the influence become obvious due to the vibration of sea water level. (3) Observation well can't reflect the variation of groundwater level instantaneous,. there is a time lag for the variation of groundwater level, for the area close to coast the time lag become obvious.

This paper linearized the Boussinesq's equation, simulate the variation of groundwater level close to observed data, although the linearized analytical solution is an approximate solution, for the average groundwater level is fixed value, so the difference is not obvious.

REFERENCES

1. Dominick, Wilkins,F.B. and Roberts,H. (1971) Mathematical model for beach groundwater fluctuations, Water Resour. Res., 7(6),1626-1635.
2. Nielsen,P. (1990) ,Tidal dynamics of water table in beaches, Water Resour. Res., 26(9),2127-2135.
3. Nielsen, P., Fenton, J.D. ,Aseervatham ,R. A. ,and Perrochet,P. (1997) Watertable waves in aquifers of intermediate depths, Adv.Water Resour.,20,37-43.
4. Carslaw, H. S., and Jaeger, J. C. (1959) Conduction of Heat in Solids, 2nd ed., Oxford University Press , London, England . 102-104 .
5. Li, L., Barry, D. A., Stagnitti F., and Parlange,J.-Y. (32000) Groundwater wave in a coastal aquifer: new governing equation including vertical effects and capillarity, Water Resour. Res., 36(2),411-420.

Table 1 、Groundwater level from Well No.7 to No.11

Water Table Data From Barrenjoey Beach, April 18-19,1989

Water Table Heights Measured in Wells

time(min)	Well-7(m)	Well-8(m)	Well-9(m)	Well-10(m)	Well-11(m)
0	0.09	0.15	0.19	0.22	0.24
30	0.09	0.15	0.19	0.21	0.24
60	0.1	0.16	0.18	0.21	0.23
90	0.12	0.17	0.18	0.2	0.22
120	0.15	0.19	0.19	0.21	0.23
150	0.19	0.22	0.22	0.23	0.23
180	0.24	0.25	0.24	0.25	0.26
210	0.34	0.30	0.28	0.28	0.29
240	0.44	0.35	0.31		0.31
270	0.53	0.40	0.34	0.33	0.33
300	0.54	0.43	0.37	0.35	0.35
330	0.53	0.45	0.39	0.37	0.37
360	0.53	0.45	0.41	0.39	0.38
390	0.46	0.44	0.42	0.4	0.38
420	0.38	0.41	0.41	0.4	0.39
450	0.34	0.38	0.39	0.39	0.38
480	0.3	0.34	0.37	0.37	0.38
510	0.26	0.30	0.33	0.35	0.36
540	0.22	0.27	0.29	0.31	0.33
570	0.2	0.24	0.27	0.29	0.31
600	0.17	0.22	0.25	0.27	0.29
630	0.15	0.20	0.24	0.26	0.27
660	0.13	0.19	0.22	0.25	0.26
690	0.12	0.17	0.21	0.23	0.25
720	0.1	0.16	0.2	0.22	0.24
750	0.1	0.16	0.19	0.22	0.23
780	0.1	0.16	0.19	0.21	0.23
810	0.11	0.16	0.18	0.21	0.23
840	0.13	0.17	0.19	0.21	0.22
870	0.16	0.19	0.19	0.21	0.23
900	0.18	0.23	0.23	0.24	0.25
930	0.27	0.27	0.25	0.26	0.27
960	0.33	0.30	0.3	0.28	0.28
990	0.42	0.35	0.31	0.31	0.31
1020	0.46	0.38	0.33	0.33	0.33
1050	0.48	0.40	0.36	0.34	0.34
1080	0.45	0.40	0.37	0.36	0.35
1110	0.39	0.39	0.38	0.37	0.35
1140	0.35	0.37	0.37	0.37	0.36
1170	0.31	0.34	0.35	0.36	0.36
1200	0.26	0.30	0.32	0.33	0.34
1230	0.23	0.27	0.29	0.31	0.32
1260	0.2	0.24	0.27	0.28	0.3
1290	0.17	0.21	0.25	0.26	0.28

1320	0.14	0.18	0.23	0.24	0.26
1350	0.11	0.17	0.21	0.24	0.25
1380	0.09	0.15	0.2	0.22	0.23
1410	0.08	0.14	0.18	0.2	0.22
1440	0.06	0.13	0.17	0.19	0.21

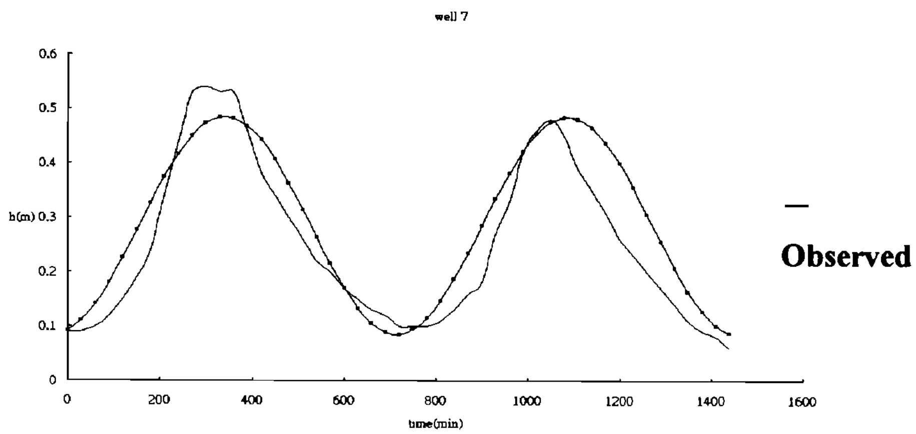

Figure1 、Observed and simulated value for well No.7

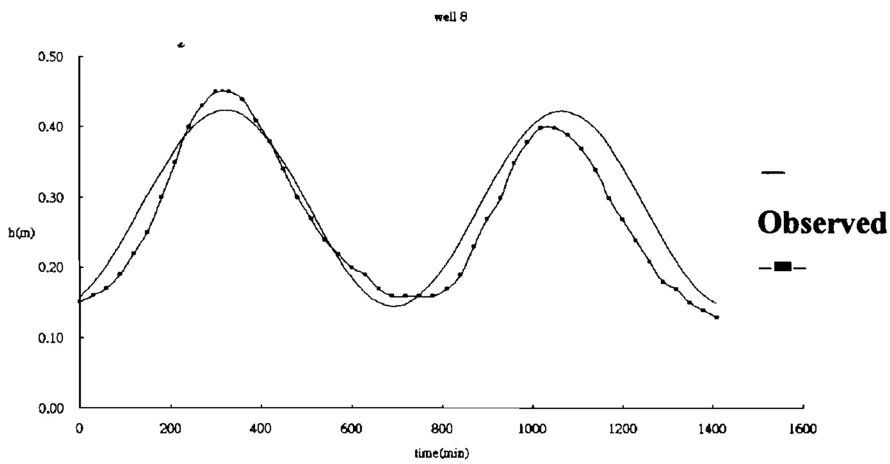

Figure2 、Observed and simulated value for well No.8

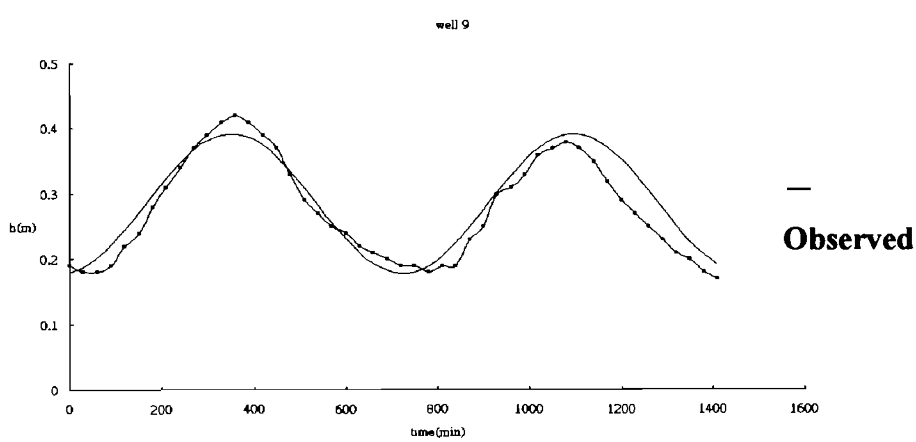

Figure3 、Observed and simulated value for well No.9

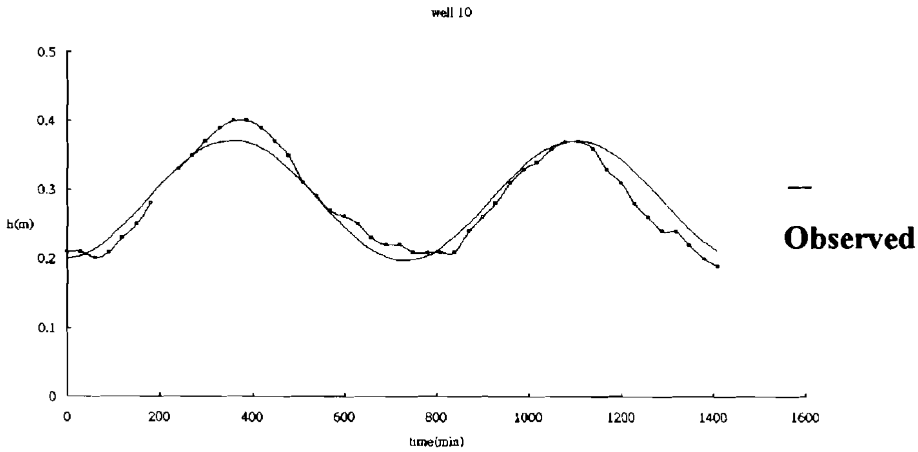

Figure4 、Observed and simulated value for well No.10

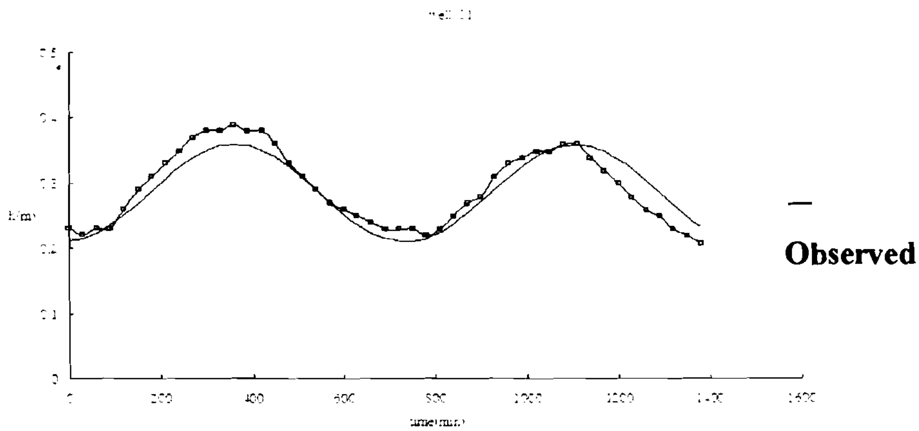

Figure5 、Observed and simulated value for well No.11

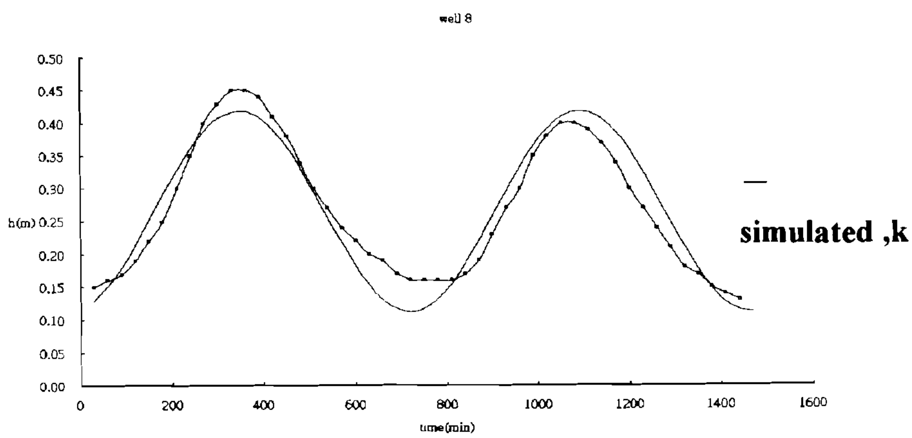

Figure6 、Observed value and analytical solution for well No.8

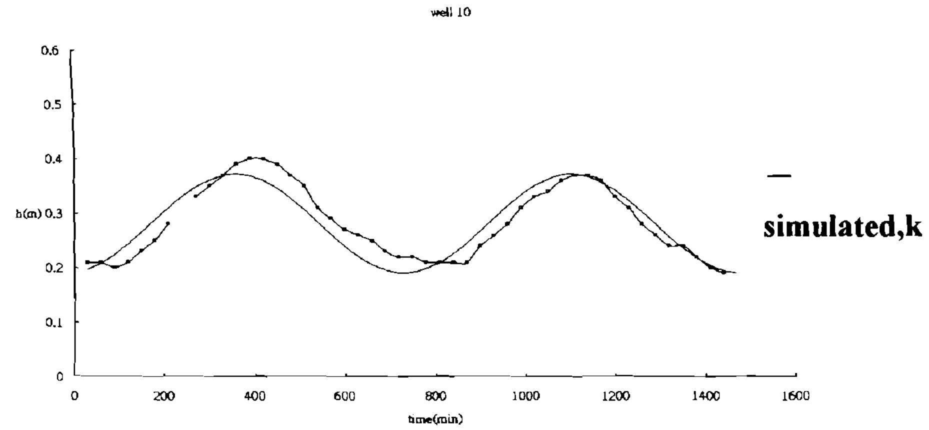

Figure7 、Observed value and analytical solution for well No.10

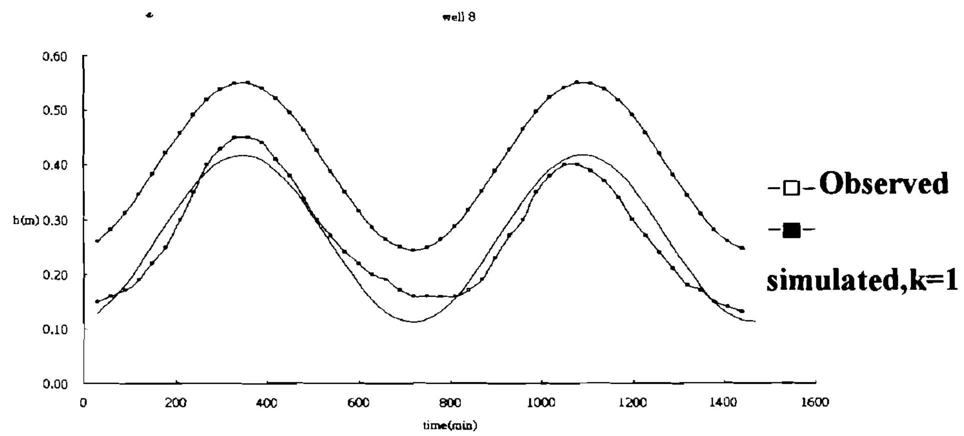

Figure8 、Observed value and analytical solution with different k for well No.8