

# Hysteretic Nature of Water Content and Pressure in Finger Flow in Unsaturated Sandy Soil

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## ABSTRACT

The stability and persistence of finger flow paths in sandy soil can determine the flow paths of subsequent infiltration events. Previous studies found that fingers persist over long periods of constant infiltration and in subsequent infiltration cycles and hysteresis could be an important factor in finger flow formation. We extend Glass et al's (1989) theory to explain finger persistence. The present theory is tested using a technique in which water content are visualized by analyze of image. The present theory and experiments show that hysteresis in the moisture characteristic relationship explains the persistence of finger flow in time. The nonuniform moisture content exists even when the potentials are equalized horizontally and that then are the result of hysteresis in the soil's pressure-saturation relationship.

Key word: finger flow, hysteresis, analyze of image, infiltration

## 1. INTRODUCTION

Water and pollutant transport in unsaturated soil is the subject of considerable current interest. Pollutants carried by infiltration water may transport vertically downward through the soil, causing groundwater contamination. For many years 1-D models have been applied for estimating the risk assessment of groundwater contamination. Prediction from this kind of models indicates that the risk to groundwater is often negligible, since pollutants have sufficient time to interact with the soil in which they reside and travel. Recent studies have revealed the existence of preferential pathways in the vadose zone, which lead water and pollutants directly and rapidly to groundwater. Groundwater pathways have been shown to underestimate its amount of pollutants reaching the groundwater table.

Preferential flow pathways are often assumed to be cracks, or other heterogeneities in field soils. In addition, field and laboratory evidence has shown flow variability for coarse grained and/or water repellent sandy soils where heterogeneities in a horizontal plane are minimal. In such soil, water and pollutants move as columnar flow paths to groundwater, rather than moving as uniform front over the entire area. This kind of preferential flow caused by wetting front instability is called fingered or finger flow. Fingered flow has been found in coarse-grained soils where the flow flux is much less than the saturated conductivity. The reduction in flux can occur by a fine texture layer over a coarse layer or simply by applying water at a low flow rate. The speed of finger flow reaching groundwater table is faster than that of stable uniform 1-D flow. Transport of pollutants by finger flow increases the risk of groundwater contamination, it terms of concentrated concentration and early arrival.

## 2. LITERATURE REVIEW

Although the existence of instability at the interface of two immiscible fluids of different densities and viscosities has long been recognized (Taylor, 1950; Saffman and Taylor, 1958), fingered flow in unsaturated soil did not attract much attention until Hill and Parlange (1972) preformed their fingered flow experiments in the laboratory. Since then, both laboratory and field experiments have been conducted (White et al., 1977; Diment and Watson, 1985; Starr et al., 1978; Glass et al., 1989c; Baker and Hillel, 1990; Selker et al., 1992a; Liu et al., 1995). At

the same period of time, theoretical analysis was attempted by Ratt (1973), Philips (1975), Parlange and Hill (1976), Diment et al., (1982), Diment And Watson (1983), Hillel and Baker (1988), Glass et al. (1989a) and Selker et al. (1992c).

In most experiments, two layered systems of fine over coarse soil were used to develop fingered flow. In those experiments often the water was ponded on the surface of the fine soil (Hill and Parlange, 1972; While et al., 1976; Glass et al., 1989c; Baker and Hillel, 1990). Since the fine soil limits the water flux entering the coarse bottom soil, the wetting front at the interface becomes unstable and fingers form in coarse bottom soil. Recent experiments have indicated that fingered flow is not limited to two layered system, also occurs in homogeneous sandy soil with flux much less than the saturated conductivity (Selker et al, 1992a). To establish the geometric and temporal pattern of finger flow in unsaturated soil is an important step in the quantitative study of finger flow (Hillel, 1987). In the literature, there have been various predictions regarding the finger width. Philips (1975) derived an analytical expression to predict finger width by using hydrodynamic stability analysis. His analysis was based on Green and Ampt model of infiltration. White et al. (1976) extended Philips's model in a general form for porous media. Parlange and Hill (1976) proposed a theoretical model to predict finger width from the wetting front instability. Glass et al. (1989a) confirmed the validity of Parlange's equation and used the scaling theory of Miller and Miller (1956) to predict the finger width for different porous media.

Although antecedent moisture condition and hysteresis were included in some of previously cited analyses, the role of hysteretic relationship of the water content and pressure in finger flow has not yet been well examined theoretically and experimentally. Raats (1973), Hillel and Baker (1988), Glass et al. (1989c) and Liu et al. (1995) indicated that hysteresis could be an important factor in finger formation. It is clear that the understanding of physical processes of finger formation is not complete without exploring possible effects of hysteresis of the water content and pressure relation. In this study, the effects of hysteresis on finger are studied through laboratory experiments observing finger formation in initial dry soil.

The development of fingers and the steady-state flow field that forms upon long-term ponded infiltration in an initially uniformly dry, fine-over-coarse-textured, layered sand system was shown qualitatively through the use of dyes and photographic documentation by Glass and Steenhuis (1984) and Glass et al. (1987). The flat, downward-moving wetting front in the top fine layer becomes unstable as it passes into the bottom coarse layer, causing the formation of fingers in the coarse layer. Three stages in the evolution of the unstable flow field were noted. The initial stage is dominated by rapid downward movement of fingers that form finger "core" areas. When supplied at a constant flow rate, fully developed fingers maintain a constant finger tip velocity and widen rapidly to a constant width as the finger tip passes. Glass et al. (1989) have quantified relationships in this stage among flow through the finger, finger width, and finger velocity. In general, the higher the flow, the wider the finger and the higher the velocity of the finger tip. The secondary stage is characterized by the persistence of finger core areas that continue to conduct most of the flow and by the slow lateral movement of wetting fronts with less moisture from finger core areas into the surrounding dry sand. A less saturated "fringe" area thus develops between the more saturated finger core areas. The lateral movement in the second stage is slow, having a time scale on the order of days, but the time scale for the downward finger growth in the first stage is much faster, on the order of minutes. The final stage is a steady-state flow field in which core and fringe areas coexist for long periods of steady infiltration. Both the second and last phases, which had not been noted by earlier experimental studies (Hill and Parlange, 1972; Diment and Watson, 1985; Glass et al., 1989, Liu et al., 1995), demonstrate the important feature of core/fringe structure formation.

In addition to demonstrating the formation of the core/fringe structure steady infiltration events, Glass et al. (1987) demonstrated the persistence of fingers from one infiltration cycle to the next. After an interruption in the water supply and drainage to field capacity, fingers form in the same locations as they did in the first cycle and have the same core areas, which continue to conduct almost all of the water. Fringe-area contribution is higher than in the first cycle, and a steady-state flow field is achieved much more rapidly. If the chamber is flooded and drained so that the initial moisture content field is made uniform, core areas are obliterated, thus emphasizing that finger persistence was not caused by heterogeneities in the porous material either in the initial pack or because of possible reorientation of grains by the initial fingers themselves.

In this paper, we present a physically based theory to explain the mechanism of finger persistence, and we verify the theory through experimentation. We develop an experimental technique to rapidly visualize the moisture content field in thin slabs of porous media. The technique is applied to carefully document the moisture content structure of fingers as they move downward and the persistence of finger structures in a second infiltration event after an interruption in water supply.

### 3. THEORY

A clue to understanding finger persistence was uncovered in an early study by Raats (1973), who discussed Tabuchi's (1961) finding that the potential of the water at a finger tip is less negative than that at the sides of the finger after the finger tip has passed. Raats stated: "It appear to me that this may be due to the fact that the capillary pressure head at the tip of the (fingers) corresponds to the point on the water content-capillary pressure head curve for wetting, while that at the stationary or receding part corresponds to the one for drying." In agreement with Raats' interpretation, we have observed that some individual pores empty a short distance behind the finger tip, thus decreasing the moisture content. When drying behind the finger tip occurs, the three stages in the development of the unstable flow field and finger persistence over repeated infiltration cycles can be explained quite simply.

Figure 1 is a sketch of a finger moving within an initially uniformly dry porous medium. Three zones within the vertically extending finger core area are shown. At the tip is a zone of high moisture content,  $\theta_t$ , very near or at the saturated value,  $\theta_s$ , followed by a zone of drying where the finger core passes from  $\theta_t$  to  $\theta_c$ , the moisture content of the finger core. Surrounding the entire core area on all sides is a narrow zone of wetting where the moisture content increases rapidly from  $\theta_i$  in the dry porous medium to the moisture content in the finger core.

Drying behind the tip decreases the matric potential and essentially halts rapid finger-core widening because of the sensitivity of the sorptivity to the potential at which the water is supplied. A uniform finger-core width only a short distance behind the finger tip is thus created. The two characteristics of the second phase of unstable flow-field development, i.e., the slow lateral widening of the fringe, compared with the rapid downward movement in the cores, and the lower moisture content in the fringe area, are direct consequences of the drying process behind the finger tip. The lateral movement of water from the finger core into the dry sand on either side of the finger takes place from a supply potential in the core that is less than what occurred when the finger tip passed. For an indefinitely long period of steady flow, lateral movement continues until the potential everywhere in the horizontal direction is the same. Because the sand in the fringe areas is becoming wet and that in the finger core areas is drying, then, as a result of hysteresis, two distinct moisture content zones coexist when the potential is equilibrated in the horizontal, thus yielding the observed third stage in unstable

flow field development. With the use of a moisture characteristic relation that includes hysteresis, the theory may be explained in more detail. Figure 2 is a plot of matric potential,  $\psi$ , versus the moisture content,  $\theta$ , for a typical sand. Considering a point on the axis of a finger, as the wet finger tip moves into dry porous medium, the wetting curve is descended to the point  $(\theta_t, \psi_t)$  very near or at  $(\theta_s, \psi_{ae})$ , the saturated value of the moisture content and water entry value of the matric potential, respectively. After the tip passes, the matric potential decreases to the value  $\psi_{ae}$  at which point the finger core begins to dry. A drying curve is thus ascended from the point  $(\theta_t, \psi_t)$  to the point  $(\theta_c, \psi_c)$ , where the moisture content of the finger core,  $\theta_c$ , is less than  $\psi_t$  and the corresponding matric potential  $\psi_c$  is less than the air entry value  $\psi_{ae}$ . On a longer time scale, the dry Sand on either side of the finger now wets by coming down the wetting curve until it reaches the matric potential  $\psi_c$ . Figure 2 shows that the moisture content in the finger core,  $\theta_c$ , and that in the fringe area,  $\theta_f$ , can be substantially different. This difference is observed in the final stage of the unstable flow-field development for infiltration into initially dry porous media. Thus, at equilibrium, two zones of different moisture context can coexist, and, because of the strong variation of the hydraulic conductivity with moisture content, almost all of the flow continues to take place through the finger cores as observed (with flux equal to the hydraulic conductivity,  $k(\theta_c)$ , in the core area).

When infiltration stops at the end of the first infiltration cycle, the moisture drains out of the finger core area, and the main drying curve is ascended to the point  $(\theta_{dc}, \psi_d)$  when the water essentially stops flowing.  $\psi_d$  varies in the vertical and offsets the change in the gravity potential with depth when flow stops. The fringe areas also drain, and the matric potential moves to the value  $\psi_d$ ; however, it follows the scanning curve shown in Figure 2 to the point  $(\theta_{df}, \psi_d)$ . In this new equilibrium, there are again two moisture content areas after drainage, and the potential is constant in the horizontal.

#### 4. EXPERIMENT

To verify the finger's moisture content model and the hysteresis in finger's fringe areas, experiment in this research use sand-box to analysis. Using gather image in finger's growth process to gain the finger's moisture content variation, making use of the hysteresis's curve is the main reason and analysis finger's flow potential distribution mechanism and flow's path is stable. Experiment divides into three parts, the first part is calibration the experiment equipment including the equipment measuring hysteresis's curve and the moisture content and image gray-value relationship; the second part is determining the moisture content's hysteresis's curve; the third part is finger flow experiment that use layed soil's structure to make the environment producing finger flow.

The process of experiment divides into three parts : (1)testing of soil's basic characteristics(2)calibration the experiment equipment(3)sand-box experiment establishment and implementation, specify as follows:

##### 4.1 Testing of Soil's Basic Characteristics

The soil applied in this experiment was of quartz stone classified into two types with respect to their grain sizes. In order to avoid confusion, the soil of bigger granular diameter was named as Soil-1 where 87.42% of its content was distributed within the range 0.59mm~

0.71mm. The soil of finer granular diameter was named as Soil-2 where 76.05% of its content was distributed within the range 0.25mm~0.297mm. Both types of the soil are of very high uniformity.

This research was to implement the experiment with the above-mentioned soils of different natures and with their differences in soil characteristics, stratum of sandstone structure was formed for constructing the environment that could produce finger flow.

## **4.2 Calibration the Experimental Equipment**

The calibration of experimental equipment includes the calibration of soil water content characteristic curve measuring instrument and the calibration of the relationship of soil water content and gray-level of image. In the measuring of soil water content characteristic curve, the water content of soil was measured by TDR (Time Domain Reflectometry). It was formed by connecting Trime-ES and Miniatur Probe P2D manufactured by IMKO Micromodultechnik GMBH. The detecting needle was diode bar type, which was applicable to perform measurement in a laboratory or job site. The instrument's applicable range was the water content within 0~95% of mass, the allowable tolerance of the instrument was within the range of  $\pm 2\%$  of mass water content. TDR applied electromagnetic wave measuring theorem to measure soil water content (Topp, 1980), by using the relationship of soil's dielectric constant and mass water content to determine the water content of the soil.

And the tension of soil water content was measured with Tensiometer. It's T5 Tensiometer manufactured by Umweltanalytische Mess-Systeme GMBH. It's applicable range was -1000hPa~850 hPa, the allowable tolerance range of the instrument was  $\pm 5$ hpa. Its structure included a porous cup; it was connected to the tension changer. By using Tensiometer and the balance of tension of the soil water content to measure the tendency capacity status of the soil water content. And the relationship of soil water content and gray-level of image was, by using digital image instrument to obtain the relationship of different soil water content and gray-level of image. The instrument for capturing digital image is the digital camera - Canon Powershot G1. Its image resolution can reach 3.34 Million pixels. The image treating software is for processing analysis with Image-pro Plus designed by Media Cybernetics.

The result obtained from the calibration of Tensiometer, Time Domain Reflectometry and the relationship of soil water content and gray-level of image was as stipulated in Table-2. The relative coefficient obtained from the result of rate setting for Tensiometer and Time Domain Reflectometry reached 0.998 and 0.995. This indicates that the experimental equipment can precisely reflect the actual tensile water head and water content. And the relationship of gray-level of image and soil water content under different illuminations also appeared in a linear relation, the obtained relationship reached within 0.91~0.94, it is deemed as highly relative.

## **4.3 Sand-box Experiment Establishment and Implementation**

### **4.3.1 Measurement of Soil Water Content Characteristic Curve**

In this research, gravel sand box mock up was applied for testing the changes of soil water content in unsaturated stratum. By applying this, the soil water content characteristic curve could be measured. Apply Time Domain Reflectometry to measure soil water content; apply Tensiometer to monitor the change rate of soil water tension. Combining the above-mentioned two, the tension and soil water content of same duration can be picked. Through the border line condition to control wet or dry status of soil to obtain the curve of soil water content hysteresis-nature as the basis for determining the change status of the analyzing finger flow's water content.

During the measurement of hysteresis experiment, Tensiometer, Time Domain Reflectometry were both connected to Data Logger. The Data Logger is 21X model Micro-Logger manufactured by Campbell Scientific INC., which is available for long-time application in monitoring and handling of experimental data; the gravel sand box used in the experiment was formed by the combination of 8mm thick acrylic plate. The gravel sand box had an inner length of 20cm, width 6cm, height 15cm. Its bottom was bored with a hole of 8mm diameter and fastened with a water control valve, and it was connected to water transmission conduit. As per the experiment's requirement, the water transmission conduit was connected to air exhauster (drainage status) or water storage container (wet status). Among them, one water storage container was connected between the air exhauster and water transmission conduit for using as water storage. In order to prevent air exhauster from absorbing water content, on right side of gravel sand box, one tiny hole of 8mm diameter and one tiny hole of 21mm diameter were bored. The 8mm tiny hole was provided for inserting Tensiometer, 21mm tiny hole was provided for inserting Time Domain Reflectometry; the bottom of gravel sand box model needed to be paved with a layer of highly permeable geotextile for preventing gravel sand and soil from entering the water transmission conduit along with the water.

The determination of moist progress was by controlling the elevation of the water storage container, to apply water level and capillarity theorem to make the soil mass wet and proceed toward saturation. Record the corresponding relationship of the water content and tensile value during the process to obtain major wet curve. The determination of dry progress was by switching the water controlling valve at the lower part of the gravel sand box on for processing water content discharge. If the water discharging speed was slow, then water pump would be connected to the water control valve at the lower part of the gravel sand box for pumping of water and air in achieving the purpose of fast drying. Record the corresponding relationship of the water content and tensile value during the process to obtain major dry curve.

Moreover, the internal water content's characteristic curve applied the data of the above-mentioned dry and wet water content's characteristic curve to cooperate with the analyzing method proposed by Huang Hang-Cheng etc (2002, 2001) for further implementation. The obtained curvature parameters were as stipulated in Table-1. By using this, subsequent analysis on the changes of the soil water content and tension during the finger flow occurrence.

#### **4.3.2 Establishment and Performance of Finger Flow Gravel Sand Box Experiment**

Experimenting gravel sand box were made with 10mm thick acrylic material. The gravel sand box's interior mass was length 50cm, width 1cm, height 60cm. Rainfall device was the application of a total of 400 clinical-use needles of 1mm diameter which were placed at the bottom of the water storage box, the water source of the water storage box was filled by water pump. The experiment was for processing monitoring by rainfall device's water dropping rate  $318 \text{ cm}^3 / \text{min}$ . The arrangement of the experiment was as shown in Diagram 8. The distribution layers of soil in the gravel sand box were mainly Soil-1 of bigger diameter grain at the lower layer of the layer structure of 54cm thick. The upper layer of soil was Soil-2 of finer grain of 1.5cm thick. The above-mentioned conditions were applied for forming unstable permeability status of finger flow, and the monitoring of water content in the processing finger flow formation and developing stages.

### **5. RESULT**

In the experiment, the water content permeability test was processed with layers of soil structure. By using the picked permeating frontal surface movement image to obtain the water

content changes of the soil's permeating frontal surface. The obtained experimental result is as follows:

### **5.1 Finger Flow's Water Content Distribution and Water Content Changes**

The distribution of water content in finger flow was obtained by applying the picked finger flow image gray-level transformation. Figure 9 indicates the water content distribution of single flow conduit at the 30<sup>th</sup> sec after finger flow appeared where the flow conduit's water content mounted up to the highest at the front edge of moist frontal surface. Taking the x-section at each longitudinal location of the flow conduit, the distribution of longitudinal water content of the finger flow was obtained (Figure 10~ Figure 12). Among which, the location of Figure 10, Figure 11 were about at the central location of the flow conduit. The longitudinal water content at the location appeared the tendency of increment from upper to lower.

The tendency of the water content distribution was similar to the water content concept model (Figure 2). The two differed in the water content of the finger flow tail section was not maintained at specific water content distribution. Its distribution appeared to have changed from small toward big. However, the water content at the front edge of the moist frontal surface did not appear to have reached the saturated status of water content, the water content ranged within 0.36~0.39. Nevertheless, the degree of saturation even mounted up the figure above 90%; and the distribution of traverse water content was as shown in Figure 13. The distribution of traverse water content at each different depth appeared the tendency of decrement from the finger flow center toward two sides, and the status of rapid decrement of water content appeared at the edge of flow conduit. This water content distribution tendency was similar to the water content concept model described in this article. The two differed in the water content in the finger flow conduit was not a specific value. Water content distribution was not the planned distribution. But speaking of the overall entity, the water content concept distribution of this article could appropriately reflect the water content distribution of finger flow.

### **5.2 Finger Flow Border's Water Content Changes and Hysteresis Phenomenon**

According to the occurrence of water content concept model and hysteresis phenomenon, the water content changes of finger flow conduit border was the sort of changes as indicated in Figure 7a. Taking the experimenting flow conduit 4 as an example (Figure 14), 3 location points of length 0.2cm and width 0.2cm at flow conduit border were picked (Figure 15), the relationship of the water content changes was as shown in Figure 19. The water content of the 3 location points when  $t=55\text{sec}$  abruptly increased as the finger flow reached. Whereof, as C point located inside the flow conduit so that its water content abruptly increased up to 0.37 whereas as Point A and B located at flow conduit edge so that its water content when  $t=70\text{sec}$  were respectively increased to 0.33 and 0.24; 3 location points, after abruptly increasing their water content, all appeared decrement phenomenon. This condition was caused by the finger flow head portion that passed through the location. Whereof, C point's water content within  $t=55\sim 100\text{sec}$  had lowered from 0.37 to about 0.33. And after  $t=100\text{sec}$ , the water content could roughly be maintained around 0.33. And Point A and B's water content after draining period  $t=70\sim 100\text{sec}$ , appeared increment tendency (the source of the increased water content was the permeating water down from the top of the said location point or from the lateral diffusion of C Point). And after  $t=150\text{sec}$ , water content changes could roughly be maintained stable, Point A's water content changed around 0.18 whereas Point B's water content changed around 0.23; and after  $t=150\text{sec}$ , the water content of the 3 location points merely appeared slight change. This type of flow conduit edge zone was only 0.2cm distance apart but was able

to maintain the water content difference of 0.05 and 0.10 and the water content was not applicable for further explanation on the concept of flow from wet toward dry location.

The 3 location points merely at 0.2cm away, the reason that their water content after  $t=150\text{sec}$  could continuously maintain a change 0.05 and 0.10 should be due to the 3 location points had the tendency of reaching a balance. The concept of tendency resulted from single water content's characteristic curve could not provide a reasonable explanation. We can see from Figure 16 that the 3 location points' water content appeared the phenomenon of alternating between dry and wet courses. Moreover, such kind of phenomenon was the single water content's characteristic curve and it was not able to describe the water content hysteresis effect.

In hysteresis phenomenon, different location points could form adjacent location as they possessed individual water content characteristics curve despite they had different water content but were capable of reaching of a balanced status. According to the flow conduit border's water content change drawing (Figure 7a) developed from the water content concept model and the water content change (Figure 16) obtained from this experiment in this article, we can see that, the concept of the water content change of finger flow border specified in this article could appropriately reflect the water content changes of the flow conduit edge. If certain range of water change from the experiment data was omitted, then the water content change obtained from the experiment could be indicated by the tendency as shown in the Figure 17. According to the hysteresis phenomenon of the combined tendency of water content's dry and wet changes, the 3 location points' water content characteristic curve was as shown in Figure 18. The 3 location points' tensile water head after the time  $t=150\text{sec}$  was approximately 10cm, under the said tensile water head to have reached the status of tensile balance. Therefore, water content differences were found to be 0.05 at Point C & B, and 0.10 at Point B & A.

Under the influence of the above-mentioned hysteresis phenomenon to the finger flow, although flow conduit edge location had different water content, but under the status of already reached balance, the adjacent location of the flow conduit edges were not able to generate the effect of horizontal diffusing of water content. Therefore, the width of finger flow conduit is not likely to continuously expand outward.

## 6. CONCLUSION

Layered soil construction can create finger flow, and produce to suspend at the interface, which coincide the phenomenon that Daniel Hillel and Ralph S. Baker bring up in 1988. Finger flow belong Darcy's flow,  $Re=2.21\sim 3.80$  and under four different water-injection rate, the velocity and width of finger flow increase as water-injection rate increased. This paper's experiment conclusion is completely different from the paper Hill and Parlange (1972) bring up that changing water-injection rate merely change the finger flow number. The water content distribution of finger flow show the highest water content at finger tip, and have a trend that decrease from finger's tip to finger's rear. The edge of finger flow path exist a thin area that soil water changed sharply, and flow path width does not become width rapidly because of hysteresis effect when flowing. Although the vicinity location nearby the edge of finger's flow path possess different soil water content, the vicinity location can't produce soil water content diffuse horizontally and finger flow path width is almost constant under tension equilibrium state. This phenomenon is very different from traditional concept that soil water diffuse from wet area to dry area.

In groundwater recharge, we can use layer soil to produce finger flow, and finger flow preferential velocity is much faster than general infiltration velocity, which pass unsaturated area quickly and increase groundwater recharge.



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