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Optimal Operation of Irrigation Canal Systems Using Nonlinear Programming - Dynamic Simulation Model

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Abstract
Shortages of water caused by competing uses of limited water supplies in many areas of the world emphasize the need for more flexible, more beneficial and higher quality operation of irrigation conveyance systems. A non-linear programming (NLP) model combined with a dynamic simulation model which simulates irrigation conveyance system operations and hydraulics was developed to assist operators or watermasters in making optimum operational decisions. The method of Nelder and Mead was chosen as the optimization technique to minimize water loss and satisfy water demand. A non-linear programming model linked with the Irrigation Conveyance System Simulation Model promises to overcome most of the limitations associated with other optimization techniques. The computer model can be used to operate conveyance systems at maximum efficiency allowing the prediction and evaluation of the sequence and timing of operations which must be made to achieve optimal water allocations if they are achievable at all. The computer model was successfully demonstrated on typical multi-reach canals of southern Alberta, Canada.

Introduction

Irrigation conveyance systems are used to transfer water for irrigation of farmlands with the objective of helping farms to increase crop yields. They may also be used to provide water for municipal and industrial use, to carry storm and snowmelt runoff to natural drainage channels, to collect water from several independent sources into a single supply, and to supply water for fish. The objective of operating an irrigation conveyance system is to serve the above purposes as efficiently and economically as possible. Generally, the basic operational requirements for irrigation conveyance systems include:

1. deliver water to intended users,
2. transfer specified amounts of water from upstream source(s) to downstream users at the required times.
3. match the inlet supply to the downstream requirements, and
4. operate within the allowable constraints.

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Shortages of water caused by competing uses of limited water supplies in many areas of the world emphasize the need for more flexible, more beneficial and higher quality operation of irrigation conveyance systems. Irrigation canal systems, even apparently simple ones, may be very complex to operate. Even so, most canal systems are operated using techniques which rely solely on operator's judgement and experience. This usually results in low conveyance efficiency and poorly satisfied farm water demands. System control methods which use optimization techniques to assist in making operational decisions have been developed which allow the control of an entire irrigation system from one control centre. Examples are: Dynamic Regulation described by Rogier et al (1987); DYN method presented by Filipovic and Milosevic (1989) and LP operation reported by Boman and Hill (1989). Each of these techniques have several limitations which restrict their application and their utility particularly if only used to predict desired flow conditions after operations have been completed. Non-linear programming linked with a dynamic simulation model which simulates irrigation conveyance system operation and hydraulics, promises to overcome most of the limitations associated with other optimization techniques. The NLP computer model can be used to provide information on the operation of conveyance systems to achieve maximum efficiency by allowing the prediction and evaluation of the sequence and timing of operations which must be made to achieve optimal water allocations if they are achievable at all.

The objectives of this research is to develop a non-linear programming (NLP) computer model, which could optimize the operation of irrigation conveyance systems and to link the NLP model with the Irrigation Conveyance System Simulation (ICSS) model (Manz and Schaalje, 1992) to improve the performance of irrigation conveyance systems including the water delivery service to consumers.

Description of non - linear programming technique

A nonlinear programming (NLP) Model was used to assist in making decisions for the operation of an irrigation conveyance system. The objective of applying NLP is to try to maximize overall system efficiency while providing the best service to the water consumers (Lin, 1991). Nonlinear programming was selected as the optimization technique for making operational decisions because the technique permits an exact description and solution of the optimization problem and is very flexible in application.

The use of NLP is demonstrated on a multi-reach manually operated canal with several farm deliveries along its length. The model minimizes the sum of the squares of the difference between inlet release and water deliveries, the sum of the squares of the differences between farm water demands and farm deliveries, and the square of the difference between desired quantity of spill and actual spill from the last reach.

The model solves for the optimum releases (such as: optimum farm delivery rate and optimal inflow rate) for each control structure throughout the system.

The non-linear programming model of multi-reach canals was first described by Lin (1991). The objective function is written as:

$$Minimize\sum_{r=1}^{N}[(Q_{inr}-Q_{fdr}-Q_{outr}-A_r)^2+(Q_{fdr}-B_r)^2+(Q_{outn}-C_n)^2] \quad (1)$$

Where
Q_{inr} = inlet release for the r'th reach (m^3/s);
Q_{fdr} = farm outlet release from the r'th reach (m^3/s);
Q_{outr} = outlet release from the r'th reach (m^3/s);
Q_{outn} = outlet release from last reach (m^3/s); and
N = total number of reachs.

A_r, B_r and C_n are control variables defined by following general relations:

$$A_r=f(Q_{lossr}) \quad (2)$$

Where Q_{lossr} = total water loss for the r'th reach (m^3/s);

$$B_r = f(Q_{FR}, Q^o_{FR}, TO, TNOP,) \tag{3}$$

Where Q_{FR} = farm water demand for the r'th reach (m^3/s);
Q^o_{FR} = update farm delivery for the r'th reach (m^3/s);
TO = update operational time (s); and
TNOP = the time between operations (s).

$$C_n = f(Q_{DN},\ Q^0_{DN},\ TO,\ TNOP) \tag{4}$$

Where Q_{DN} = downstream requirement for last reach (m^3/s);
Q^o_{DN} = update downstream delivery for last reach (m^3/s).

Outlet release from the r'th reach is equal to inlet release for the (r+1)'th reach, that is:

$$Q_{outr} = Q_{in(r+1)} \tag{5}$$

The specific functions used to compute the control variables are:

$$A_r = \sum_{i=1}^{n} Q_{lossri} \tag{6}$$

Where Q_{lossri} = update the i'th type water loss such as seepage loss for the r'th reach (m^3/s);

$$B_r = (Q_{FR} - Q^o_{FR})(1 - e^{-TO/TNOP}) + Q^o_{FR} \tag{7}$$

$$C_n = (Q_{DN} - Q^o_{DN})(1 - e^{-TO/TNOP}) + Q^o_{DN} \tag{8}$$

Optimal control, which minimizes the criteria of Eq.1, is determined by the method of Nelder and Mead. The flow chart of the NLP solution algorithm is shown in Figure 1.

The calculation procedure used is described as follows:

1. Starting with (n+1) top points $x_i^{(k)} = (x_{i1}^{(k)}, x_{i2}^{(k)} \ldots\ldots, x_{ij}^{(k)} \ldots\ldots x_{in}^{(k)})^T$ (i=1,2,.....n+1; k=0,1,.....) and find the value of objective function in point $x_i^{(k)}$.
2. Find maximum value of the objective function $f(x_h^{(k)})$ and minimum value of objective function $f(x_l^{(k)})$. That is,

$$f(x_h^{(k)}) = \max f(x_i^{(k)}) \quad (1 \le i \le n+1) \tag{9}$$

$$f(x_l^{(k)}) = \min f(x_i^{(k)}) \quad (1 \le i \le n+1) \tag{10}$$

3. Find the centroid of all top points except $x_h^{(k)}$. Let this be $x_{n+2}^{(k)}$ and evaluate $f_{n+2}^{(k)}$, that is,
4. It would seem reasonable to try to move away from $x_h^{(k)}$. We reflect $x_h^{(k)}$ through centroid $x_{n+2}^{(k)}$ to find $x_{n+3}^{(k)}$ and $f_{n+3}^{(k)}$, that is,
   where α is the reflection coefficient, $\alpha > 0$;

$$x_{n+2j}^{(k)} = \frac{1}{n}\left(\sum_{i=1}^{N+1} x_{ij}^{(k)} - x_{hj}^{(k)}\right) \tag{11}$$

$$f_{n+2}^{(k)} = f(x_{n+2}^{(k)})$$

$$x_{n+3}^{(k)} = x_{n+2}^{(k)} + \alpha(x_{n+2}^{(k)} - x_h^{(k)}) \tag{12}$$

$$f_{n+3}^{(k)} = f(x_{n+3}^{(k)}) \tag{13}$$

5. Now compare $f_{n+3}^{(k)}$ with $f_l^{(k)}$
   a) If $f(x_{n+3}^{(k)})$ is not greater than $f(x_l^{(k)})$ the lowest function value has been obtained. The direction from $x_{n+2}^{(k)}$ to $x_{n+3}^{(k)}$ appears to be a good one to move along. Therefore we make an expansion in this direction to find $x_{n+4}^{(k)}$ and evaluate $f_{n+4}^{(k)}$, that is, $(x_{n+3}^{(k)} - x_{n+2}^{(k)})$ will be expanded using the following

relation:,

$$x_{n+4}^{(k)} = x_{n+2}^{(k)} + \gamma(x_{n+3}^{(k)} - x_{n+2}^{(k)}) \tag{14}$$

where γ is the expansion coefficient.

$$f_{n+4}^{(k)} = f(x_{n+4}^{(k)}) \tag{15}$$

   i) If $f(x_{n+4}^{(k)}) < f(x_l^{(k)})$ replace $x_{h(k)}$ by $x_{n+4}^{(k)}$ and check the (n+1) points of the simplex for convergence to the minimum (see step 9) and if not return to step 2.
   ii) If $f(x_{n+4}^{(k)}) > f(x_l^{(k)})$ we abandon $x_{n+4}^{(k)}$ because we have evidently move too far in the direction $x_{n+2}^{(k)}$ to $x_{n+3}^{(k)}$. Instead replace $x_h^{(k)}$ by $x_{n+3}^{(k)}$ ($x_h^{(k)} = x_{n+3}^{(k)}$) which now give a important check for convergence and if not return to step 2.

   b) If $f_{n+3}^{(k)} > f_l^{(k)}$ but $f_{n+2}^{(k)} < f_g^{(k)}$(the second maximum value of function), $x_{n+3}^{(k)}$ is an improved on the two worst points of the simplex and we replace $x_h^{(k)}$ by $x_{n+3}^{(k)}$, test for convergence and if not return to step 2.
   c) If $f_{n+3}^{(k)} > f_l^{(k)}$ and $f_{n+3}^{(k)} > f_g^{(k)}$ proceed to next step.
6. Now compare $f_{n+3}^{(k)}$ and $f_h^{(k)}$
   a) If $f_{n+3}^{(k)}$ is greater than $f_h^{(k)}$. It would appear we have moved too far in the $x_h^{(k)}$ to $x_{n+2}^{(k)}$. We try to rectify this by finding $x_{n+5}^{(k)}$(and then $f_{n+5}^{(k)}$) by a compression step. We proceed directly to the compression and find $x_{n+5}^{(k)}$ from

$$x_{n+5}^{(k)} = x_{n+2}^{(k)} + \beta(x_h^{(k)} - x_{n+2}^{(k)}) \tag{16}$$

where β is the compression coefficient.

$$f_{n+5}^{(k)} = f(x_{n+5}^{(k)}) \tag{17}$$

   b) If $f_{n+3}^{(k)}$ is less than $f_h^{(k)}$, first replace $x_h^{(k)}$ by $x_{n+2}^{(k)}$ and then compression. Thus find $x_{n+5}^{(k)}$ and f_{n+5} from

$$x_{n+5}^{(k)} = x_{n+2}^{(k)} + \beta(x_{n+3}^{(k)} - x_{n+2}^{(k)}) \tag{18}$$

$$f_{n+5}^{(k)}=f(x_{n+5}^{(k)}) \tag{19}$$

7. Now compare $f_{n+5}^{(k)}$ and $f_h^{(k)}$
   a) If $f_{n+5}^{(k)}$ is less than $f_h^{(k)}$ replace $x_h^{(k)}$ by $x_{n+5}^{(k)}$, check for convergence and if not return to step 2.
   b) If $f_{n+5}^{(k)}$ is great than $f_h^{(k)}$ it would appear that all our efforts to find a value which is less than $f_h^{(k)}$ has been failed so move to step 8.
8. At this step we contract the size of the simplex by halving the distance of each point of the simplex from $x_l^{(k)}$ the point generating the minimum function value. So proceed to the contraction and find $x_i^{(k+1)}$

$$x_i^{(k+1)}=0.5(x_i^{(k)}+x_l^{(k)}) \quad (i=1,2,3,\ldots\ldots,n+1) \tag{20}$$

$$f_i^{(k+1)}=f_i^{(k+1)} \tag{21}$$

9. Solution is achieved if

$$\left[\frac{1}{n+1}\sum_{i=1}^{n+1}[f(x_i^{(k)})-f(x_{n+2}^{(k)})]^2\right]^{1/2}\leq FT \tag{22}$$

Where FT is the error tolerance and is usually a very small positive value.

If above condition is satisfied, the variable multi-planes calculation stops. If above condition is not satisfied, the calculation will be continued.

10. The standard (n(n+1) matrix) is often used in the initialization of Nelder and Mead's method. This is described as follows:

$$D=\begin{vmatrix} 0 & d_2 & d_1 & \cdots\cdots & d_1 \\ 0 & d_1 & d_2 & \cdots\cdots & d_1 \\ \cdots & \cdots & \cdots & \cdots\cdots & \cdots \\ 0 & d_1 & d_1 & \cdots\cdots & d_2 \end{vmatrix} \tag{23}$$

$$d_1=\frac{d}{n\sqrt{2}}(\sqrt{n+1}-n-1) \tag{24}$$

$$d_2=\frac{d}{n\sqrt{2}}(\sqrt{n+1}-1) \tag{25}$$

where d is the maximum distance between two equidistant points of the initial simplex matrix.

If x° is the initial estimate value of the minimum point, then the program given takes an initial point from :

$(i=1,2,\ldots\ldots,n+1;\ j=1,2,\ldots\ldots,n)$

where D_{ij} is the element of the i'th row, j'th column in matrix D.

$$x_{ij}^{(0)} = x_{0j} + D_{ij} \qquad (26)$$

The accuracy, convergency and stability characteristics of the method of Nelder and Mead used as the optimization technique have been thoroughly investigated, Lin (1991). The ability to compute an optimal solution, computer system characteristics withstanding, is a function of four parameters, the reflection coefficient, the expansion coefficient, the compression coefficient and the error tolerance. To evaluate the relative significance of these parameters on the ability of the NLP algorithm to produce required solutions numerical experiments were performed. A separate NLP computer model was used to evaluate how the NLP theoretical model could be used to generate optimal operational decisions for the types of canal to be considered. The model developed permits the sensitivity analysis of the various coefficients affecting the NLP performance. For these analysis it was not necessary to simulate the actual hydraulics, hydrology and operation of the canal. Instead only initial steady state conditions need to be specified and reasonable operational objectives posed. The method has very good stability and reasonable convergency characteristics. The quality of numerical solution is significantly affected by the reflection coefficient and error tolerance. The reflection coefficient may be varied from 0.5 to 1.0, the error tolerance must be less than 0.001 and the compression coefficient may be varied from 0.25 to 0.95. The expansion coefficient only slightly affects the NLP calculation. The four parameters may be varied but are normally kept constant (reflection coefficient is equal to 0.50, compression coefficient is equal to 0.5, expansion coefficient is equal to 2.0 and error tolerance is less than 0.001) to maximum solution accuracy.

Non - linear programming - Dynamic simulation model

The framework chosen for the evaluation of the NLP Model was the Irrigation Conveyance System Simulation (ICSS) model; described by Manz and Schaalje (1992). The ICSS Model was designed to provide the model user with maximum flexibility in the inclusion, selection, and application of algorithms which may be used to simulate the physical, environmental and operational characteristics of the irrigation conveyance system. The NLP Model was linked with the ICSS Model and is called the NLP.ICSS Model.

System performance evaluation

The purpose of the system performance descriptor is to evaluate the timeliness and precision of delivery not only through farm outlets but also through the canal outlets. The system performance descriptor, P_s, first described by Sharma et al (1991) has been modified for the purposes of this study. Mathematically, the concept of system performance can be written as:

$$P_s = 1 - e_{sl}$$

$$e_{sl}^2 = \frac{\sum_{r=1}^{N} [Q_r(x) - Q_a(x)]^2 + [Q_o(x) - \sum_{r=1}^{N} Q_a(x)]^2}{\sum_{r=1}^{N} Q_r^2(x) + Q_o^2(x)} \qquad (27)$$

where e^2_{sl} = a scale -free measure of total relative error;
$Q_r(x)$ = the required amount of water in the r reach;
$Q_a(x)$ = the actual delivered amount in the r reach;
$Q_o(x)$ = total inflow into canal; and
N = total number of reaches.

The mean square prediction error formula as described by Equation 27 provides an estimate of the total error in water distribution relative to the farmer's demands or other demands for water, and the total relative water delivery loss. System performance is used to evaluate the quality of combined irrigation conveyance system operation and management practices. If $P_s = 1$, the system performance is perfect. If P_s is close to 0, the system performance is very poor.

Demonstration

Canal Description

The canal simulated is the smallest typical representative of the networks in South Alberta under consideration. A typical multi-reach canal is shown in Figure 2. The canal may be divided to four reaches and the length of each reach is 500 m. The water inflow into the canal is controlled by the manually operated inlet structure. The structure # 2 is a manually operated gated orifice with a free outlet. Structure # 3 and # 4 are manually operated gated orifices with free outlet located immediately upstream of manually operated vertical check-drop structures. Check height is adjusted using boards of fixed dimension. The canal is earthlined with trapezoidal cross-section, bottom width is 0.5 m, side slope is 3:1 and longitudinal slope is 0.001. Manning's roughness is 0.025. The Moritz coefficient used to describe seepage loss as a function of depth was taken as 0.34.

Operational Objectives

The operational objectives were to deliver water through the farm outlets to satisfy delivery requirements and to minimize water delivery losses. The delivery schedule is shown in Table 1.

Table 1 Delivery schedule

Schedule	Duration of delivery (Hours)	Farm delivery requirement (m^3/s) Turnout #1	Turnout #2	Turnout #3
#1	12	0.113	0.113	0.113
#2	12	0.113	0.071	0.113
#3	8	0.043	0.028	0.142
#4	8	0.000	0.113	0.057
#5	8	0.113	0.000	0.113

Experimental Demonstration

All operations, including timing and magnitude of change, were performed as per instructions from the model. The time between operations was 0.5 hour and the duration of the total operational procedure was 48 hours. Farm deliveries plus or minus 20% of required were considered acceptable. If the farm delivery was less than 80% of required, all of the water was considered spilled. If the farm delivery was greater than 120% of required, the portion in excess of 120% was considered spilled. All operations started 0.05 hour after initialization. The NLP component of the model provides instructions on how to operate the ICSS component of the model to satisfy the farm water demands. The time between operations are specified by the model user.

The NLP.ICSS Model was used to simulate the water deliveries indicated in Table 1. Simulations were performed using four different times between operations; 0.1, 0.5, 1.0 and 2.0 hours. Typical changes in actual (simulated) canal inflow, called the updated inflow, and the optimal inflow are shown in Figure 3. The actual and optimal inflows are virtually the same. Similarly, Figures 4, 5 and 6 show how farm water demand, actual (update) delivery to the farms and optimal delivery to the farmer vary with time. Water depth is also shown to indicate the variation in water depth at the turnouts which must be accommodated by the manual operation of both check and turnout to maintain satisfactory water deliveries. It is apparent that the actual (update) delivery is very similar to the optimal delivery which is identical to the farm water demand.

The simulation performance summary is shown in Table 2. The physical conveyance efficiency considers only seepage losses while conveyance efficiency considers both seepage loss and operational spill. System performance is defined by Equation 27. It is apparent that all of the efficiencies are high; however, there is a clear downward trend as the time between operations increase. The maximum efficiencies that this canal can achieve are attained with time between operations of 0.1 hour or less. These maximum efficiencies can be used as the basis for comparative evaluation of other (more practical) operational techniques using the same canal and hydraulic control structures.

Table 2 Summary of numerical experiments.

Time between operations (hours)	0.1	0.5	1.0	2.0
Total inflow (m^3)	41558.5	42095.0	42460.2	43255.8
Total farm delivery (m^3)	39946.3	40363.2	40137.5	39256.8
Total useful farm delivery (m^3)	39235.8	39355.5	39191.4	37716.2
Total farm spill (m^3)	710.5	1007.7	946.1	1540.6
Total operational spill (m^3)	329.3	466.3	1028.7	2645.8
Total seepage loss (m^3)	948.6	962.1	980.6	1016.3
Change in canal storage (m^3)	277.5	286.2	296.0	318.2
Error in water balance(%)	-0.1365	-0.0404	-0.0448	-0.0452
Conveyance efficiency (%)	96.121	95.884	94.529	90.755
Physical conveyance efficiency (%)	97.719	97.713	97.692	97.651
System performance (%)	99.605	99.545	99.434	99.067

Conclusions

The NLP.ICSS Model was demonstrated on a small but typical canal in southern Alberta, Canada. Experience gained with this application indicates that the NLP.ICSS Model may be configured to consider a wide range of physical configuration and operational objectives and constraints.

The nonlinear programming (NLP) model used in conjunction with the ICSS Model defines the optimal performance for a particular canal configuration. These models are embodied in a program named NLP.ICSS. It is unlikely that the optimal performance predicted by NLP.ICSS is achievable in field operated manually and/or automatically controlled conveyance systems. However, the optimal operation predicted by NLP.ICSS gives operators and managers of irrigation conveyance systems a baseline to measure actual canal operations from. Due to the nature of open channel conveyance system operations (i.e. travel time, seepage, storage, spill, etc.) the measure of actual performance as compared to optimal performance of a particular irrigation canal system will provide more realistic expectations.

The NLP.ICSS model used appropriately may provide operators of manual control systems guidelines, with which to develop manual operational procedures and a basis for evaluation of the effectiveness of these procedures.

Automatic control/remote supervisory control systems may also benefit from the use of the NLP.ICSS Model. As automatic/remote supervisory controlled systems may operate with a frequency necessary for the NLP.ICSS Model; operations of canal systems near the predicted optimal efficiency may be realized.

Research is in progress to extend the model to consider canal networks, conjunctive water supply (surface and groundwater), and to demonstrate the utility of the model on a broad range of relevant applications.

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