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Optimisation of Water Allocation in Canal Systems of ChenGai Irrigation Area

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Abstract
This paper developed an optimal crop planting pattern and water allocation of canal system based on the system enginerring principle and method. The objectives are the maximum irrigation net benefit and minimum irrigation rotation time. A case of study has been completed according to the practical situation of the north main canal of ChenGai Irrigation Area, the results of the optimal crop pattern and water allocation are satisfied. The practice has proved that the model has great significance in water saving, crop yield increasing and inhancing scientific management level of irrigation area.

1 Introduction

ChenGai Yellow River Irrigation Area is situated in Liangshan county of Shandong province with the controlled area of 542 sq kilometres and farmland 38 000 ha, among which, 20 000 ha is gravity irrigated area, 18 000 ha lift irrigation area. Main crops are wheat, corn, bean, as well as some other food grains such as millet, sweet potatoes, sorghum, etc...

Irrigation area is situated in warm temperature zone, belonging to the continental monsoon climate. The average annual precipitation is only 646.8 mm, but evaporation reachs 1365.5 mm, which features the semiarid area. Besides, the distribution of rainfall is extremly uneven in time and space, which result in serious water shortage. The source of water in this irrigated area are mainly from the Yellow River and exploiting ground water, the annual diversion water volume from Yellow River is 150 Million M3. The annual exploitation of ground water is about 105 Million M3.

There is not a set of reasonable irrigation regulation because of restrictions of the project facilities and the management level in irrigated area. The utilisation of water resources isn't reasonable. The waste of the water resources is very serious, the irrigated rotation is very long. The benefit has not been brought into full play. Therefore, it is urgent necessary to carry out optimum water allocation research in ChenGai Yellow River irrigated Area according to present situation by using system engineering methodology.

The present research is based on the North Main canal system in gravity irrigation area which include 18 branch canals, see Table 1. To establish mathematical model and achieve the optimum crop proportion and the optimum water allocation scheme of the minimum irrigation rotation the satisified results have been obtained from calculation and analysis. The results many be extended into the whole irrigation area.

2 The optimum model of the best crop pattern

In order to utilize fully the water resources, to solve reasonably the contradiction between water supply and requirement and reach the maximum benefit, the optimum crop pattern covered by each branch canal must be made. The linear programming is adopted in the present study to establish the optimum model of every branch canal separately.

Table 1 The parameter statistics of the North main canal systems in ChenGai Irrigation Area

Canals	Length (M)	Irrigated Area (ha)	Flow Capacity (M^3/S)	Distance to the mouth of Main Canal (M)
North Main Canal	10100	1018.867	2.50	0
Branch 1	1300	52.000	0.42	870
2	1279	61.400	0.37	1150
3	1262	49.400	0.37	1740
4	1260	67.133	0.37	2410
5	1307	45.733	0.37	3100
6	1308	41.667	0.20	(3470
7	1340	66.000	0.42	Lined with Concrete(3990
8	1389	69.333	0.42	4670
9	1420	53.333	0.37	5200
10	600	27.000	0.37	5900
11	450	20.267	0.37	6250
12	1420	46.867	0.42	6730
13	1390	91.733	0.42	7220
14	1320	82.200	0.67	8050
15	620	14.267	0.37	8440
16	620	44.000	0.37	8700
17	1350	49.667	0.42	9420
18	790	71.867	0.37	10100

2.1 The selection of decision variables

There are five kinds of crops in ChenGai Yellow River irrigated Area. That are wheat, corn, bean, cotton and others food grains. In order to determine the optimum different crop area irrigated by every branch canal, it should choose the different crop area irrigated by every branch canal as Decision variables (parameters). Therefore, the selected Decision variables are as follows :

Aij = the optimum crop area of the cropj in the ith branch canal unit : ha
i = 1, 2, ..., 18
j = 1, 2, ..., 5, in which, 1 stands for wheat, 2 stands for corn, 3 stands for bean, 4 stands for cotton, 5 stands for other food grains.

2.2 The design of objective function

In this paper, the objective function should achieve the maximum net benefit. Its mathematic formula as follows :

$$MaxZ = \varepsilon \sum AijPj \Delta dj - E \sum Aij$$

in the formula : ε ~ irrigation benefit allocation coefficient (0.4 in the study)

Pj ~ the price of the cropj, unit : \$/kg (see Table 2)

Δdj ~ the increased output of cropj through irrigation, unit : kg/ha

E ~ the annual operational fee in gravity irrigated area (13.6 = \$/ha)

N ~ the sort of crop (5 in the study)

i ~ the number, it's 1, 2, ... 18

j ~ the crop number, it's 1, 2, ... 5

Table 2 The situation of the crops in the North main canal systems

CROPS	WHEAT	CORN	BEAN	COTTON	OTHERS
The max.crop pattern	0.900	0.60	0.200	0.25	0.15
The min.crop pattern	0.800	0.50	0.120	0.11	0.11
Gross duty (M^3/ha times)	5.667	5.00	4.667	5.00	4.00
Growth period (days)	255.000	110.00	120.000	165.00	100.00
Unit price (\$/kg)	0.171	0.12	0.236	1091.00	
Unit output 1965 (kg/ha)	763.000	1237.50	682.500	165.00	
Unit output 1989 (kg/ha)	4590.000	4875.00	2040.000	750.00	
Increment (kg/ha)	3825.000	3637.50	1357.500	585.00	
Increment between irrigation and unirrigation (\$/ha)	261.490	5281.65	128.350	255.27	
Irrigation times	(1)26/11-05/12 (2)10/03-19/03 (3)08/04-17/04 (4)11/05-20/05	(6)21/05-28/05 (8)25/06-30/06	(6)21/05-28/05	(5)03/04-07/04 (7)21/06-24/06	(6)21/05-28/05 (8)25/06-30/06

2.3 The restrained conditions

In the calculation of the optimum crop area, besides the maximum objective function. The relevant restrained conditions should be included, which covers the restraint of total area, the restraint of the largest and smallest crop areas and the restraint of water volume. They are as follows :

2.3.a The restraint of total area

The crop area irrigated by per branch canal should be equal to or smaller than the total area irrigated by total branch canals. Considering the interplanting of wheat, corn, bean and the other food grains, the total planting area of wheat and cotton shouldn't be larger than the total area controled by the main canal systems, the total planting area of corn, bean and the other food grains should not be larger than total area controled by main canal systems either, that is :

$$Aij + Ai4 \leq Ai \text{ (ha)} \ldots$$

in formula : Ai ~ the area irrigated by the ith branch canal

$$\sum Aij \leq Ai \text{ (ha)} \ldots$$

2.3.b The restraint of the maximum and the minimum planting area

According to the local situation and the requirement of the contracted farm land, the areas irrigated by every branch canal have the maximum and minimum limitation to the planting area of different crops.

The maximum and minimum restraint conditions as follows :

$$Ai1 \geq 0.8Ai \text{ (ha)}$$

$$Ai1 \leq 0.9Ai \text{ (ha)}$$

$$Ai2 \geq 0.5Ai \text{ (ha)}$$

$$Ai2 \leq 0.6Ai \text{ (ha)}$$

$$Ai3 \geq 0.12Ai \text{ (ha)}$$

$$Ai3 \leq 0.20Ai \text{ (ha)}$$

$$Ai4 \geq 0.11Ai \text{ (ha)}$$

$$Ai4 \leq 0.25Ai \text{ (ha)}$$

$$Ai5 \geq 0.11Ai \text{ (ha)}$$

$$Ai5 \leq 0.15Ai \text{ (ha)}$$

2.3.c Water volume restraint

The irrigation area requires water supply for 8 times yearly, specific irrigation time can be seen in Table 2. Every irrigation rotation should be finished within the required time, otherwise, the plant yield is influenced. The volume of water use can be obtained through the calculation of crop area and the related irrigation duty (see Table 2).

The volume of water supply have been obtained according to the maximum volume of water diversion in the North main canal. The restraint condition of the water volume as follows :

$$A_{i1}M_{k1} \leq \eta_m \eta_{bi} \frac{Q_M \cdot A_i}{F} \times d_k \times 24 \times 3600 \text{ (m3)}$$

in formula : M_{k1} ~ Irrigation duty for crop 1 in k irrigation rotation (m3/ha) K = 1, 2, 3 ...

η_m ~ water use coefficient in the main canal, it's 0.92

η_{bi} ~ the water use coefficient in No. i branch canal
$\eta_{bb} = 0.99$ $\eta_{b7} = 0.99$
$\eta_{bi} = 0.71$ for other branch canals

dk ~ the total days of the k irrigation rotation

Q_m ~ the maximum diversion flow in the North main canal (2.5 m3/s)

F ~ the irrigated area controlled by the North main canal, it's 1018.867 ha

The influence of soil moisture content has not been considered in above formula for it is very difficult to forcast long term soil moisture content. Therefore, the present paper select the average irrigation duty to analyse crop pattern.

Because the first four irrigation rotation are the wheat irrigation, conditions of irrigation are equal to the conditions of water requirement.

So they can be merged into the same equation, that is :

$$A_{i4}M_{5,4} \leq \eta_m \eta_{bi} \frac{Q_M \cdot A_i}{F} \times d_5 \times 24 \times 3600 \text{ (m3)}$$

$$A_{1,2}M_{6,2} + A_{i3}M_{6,3} + A_{i5}M_{6,5} \leq \eta_m \eta_{bi} \frac{Q_M \cdot A_i}{F} \times d_6 \times 24 \times 3600 \text{ (m3)}$$

$$A_4M_{7,4} \leq \eta_m \eta_{bi} \frac{Q_M \cdot A_i}{F} \times d_7 \times 24 \times 3600 \text{ (m3)}$$

$$A_{i2}M_{8,2} + A_{i5}M_{8,5} \leq \eta_m \eta_{bi} \frac{Q_M \cdot A_i}{F} \times d_8 \times 24 \times 3600$$

The calculation result is listed in Table 3. All of calculations have been compeleted by IBM/PC computer. The optimum crop areas controlled by the branch canals have been obtained through calculations, and the objective function values have been printed (OBJ.FUNC). The form of the dailogue between the computer and person are adopted. In calculation, the key of k (number of branch canals), Fk (area controlled by branch canals) G2 (water use coefficient in branch canals), QM (the maximum flow capacity in main canal), should be inputted.

Table 3 The results of the optimum crop area unit : ha

CROPS	1	2	3	4	5	OBJ.FUNC. ($)
Canals 1	46.280	31.20	7.280	5.720	7.10	20760.218180
2	54.646	36.84	8.596	6.754	9.21	24513.018180
3	43.966	29.64	6.916	5.434	7.41	19722.200000
4	59.749	40.28	9.399	7.385	10.07	26801.963640
5	40.703	27.44	6.403	5.031	6.86	18258.345450
6	54.883	37.00	8.633	6.783	9.25	24619.472730
7	58.740	39.60	9.240	7.260	9.90	26349.490910
8	61.707	41.60	9.707	7.627	10.40	27680.272730
9	47.467	32.00	7.467	5.867	8.00	21292.527270
10	24.030	16.20	3.780	2.970	4.05	10779.340000
11	18.037	12.16	2.837	2.229	3.04	8091.158182
12	41.711	28.12	6.561	5.155	7.03	18710.800000
13	81.643	55.04	12.843	10.091	13.76	36623.145450
14	73.158	49.32	11.508	9.042	12.33	32817.109090
15	12.697	8.56	1.997	1.569	2.14	5695.750909
16	39.160	26.40	6.160	4.840	6.60	17566.330910
17	84.253	56.80	13.253	10.413	14.20	37794.236360
18	63.961	43.12	10.061	7.905	10.78	28691.672730

3 The optimum water allocation scheme based on the minimum irrigation rotation

The trial algorithm has been taken to calculate the minimum irrigation rotation required in the North main canal irrigation systems under the condition of the optimum crop areas. The optimum water allocation has been calculated according to different branch canals, under the condition of maximum flow diversion in the North main canal, the irrigation follow on system and water diversion volume of each branch canals have been regulated to realize the shortest irrigation rotation and the optimum water allocation scheme.

3.1 Decision variables

The discharge QM in the North main canal related to the irrigation rotation have been selected as the decision variables.

3.2 The objective function

The minimum irrigation duration (Tk) related to different irrigation rotation has been selected as the objective function, that is :

$$MinTk = \frac{\sum_{i=1}^{m} \sum_{j=1}^{n} A_{ij} (M_j - W_k)}{Q_M \cdot \eta_m \cdot \eta_b \times 24 \times 3600}$$

in which : Wk ------- soil moisture content, (m3/ha)
Qm ------- the maximum discharge in the North main canal : (m3/S)

3.3.a The restrained maximum discharge

The flow diversion in the North main canal and its branch canals can't be greater than the relevant diversion abilities.

$$Qmi \leq Qm$$

in which, Qmi ------- the ith discharge diversion in the North main canal
i = 1, 2, ... (m3/s)

3.3.b The minimum discharge

In order to prevent the main and branch canal systems from silting up, the flow diversion in the main and branch canals shouldn't be less than the half of the maximum flow capacity in the related canals.

$$QR \geq Qm/2$$
$$qRi \geq qmi/2$$

in which : QR ------- the practical flow diversion in the North main canal (m3/s)
qRi ------- the practical discharge diversion in the branch canal i (m3/s)
qmi ------- the practical flow capacity in the branch canal i (m3/s)

3.3.c The restraint of irrigation follow-on system

While regulating irrigation order, it is necessary to keep the irrigation order from the downstream to upper stream in order to minimize the loss caused by evaporation and infiltration, that is : i=18, 17, ... 1.

In order to minimize the objective function and guarantee the irrigation order from downstream to upperstream, the irrigation order of branch canals have been regulated in the scope of neighbor branch canals.

3.4 Analysis on research results

The calculation results will be seen in Table 4. It shows that the flow in the North main canal reaches maximum flow capacity in the first 8 days, and the flow is greater than the half of the North main canal in the following 2 days, which can be prevent silting up in branch canals as well as control the discharge diversion at the head of branch canals convenienthly. One irrigation rotation under the optimum water allocation requires 10 days, which saves about 4 days, comparing with the 14 days of the traditional irrigation system. Therefore, the purpose of time and water saving have been achieved.

The time unit of the present research is based on days for convenient management. The time unit can be shortened for further water saving purpose, so long as the parameter Ti in the programme is changed, the relative results wil be obtained. When irrigation rotation starts, the measured W and Qm can be inputted into computer, the relative results will be achieved through calculation. The data input are realized through the type of man-computer and calculation will be carried out by IBM/PC computer.

Table 4 The study scheme of the minimum irrigation rotation

unit : m3/s

DAYS	**1**	**2**	**3**	**4**	**5**	**6**	**7**	**8**	**9**	**10**	**11**
Canals 1								0.38	0.42	0.42	
2						0.19	0.37	0.37	0.37		
3							0.33	0.37	0.37		
4							0.18	0.37	0.37	0.37	
5								0.18	0.37	0.37	
6						0.09	0.19	0.19	0.19	0.19	
7						0.42	0.42	0.42			
8				0.42	0.42	0.42	0.42				
9					0.37	0.37	0.37				
10				0.24	0.37						
11					0.30	0.37					
12		0.25	0.25	0.42							
13	0.42	0.21	0.21	0.42	0.42	0.42					
14	0.35	0.66	0.66								
15	0.37										
16	0.37	0.37	0.37								
17	0.42	0.42	0.42	0.42	0.42						
18	0.37	0.37	0.37	0.37							
Q_M	2.50	2.50	2.50	2.50	2.50	2.50	2.50	2.50	2.28	4.81	