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Decentralized Constant-Volume Control of Irrigation Canals

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Abstract
The Saint-Venant equations of open-channel flow were linearized using the Taylor series expansion around an equilibrium condition. A set of linear dynamic equations were obtained for a canal with multiple pools. The principles of optimal control and estimation theory were then applied to derive a decentralized controller and an observer for constant-volume control of irrigation canal pools in the presence of external disturbances acting on the system. Using the linearized Saint-Venant equations and an example problem with 5 pools, the performance of the control algorithm in maintaining a constant-volume in the canal pools was found to be excellent.

Introduction

The need for real-time monitoring and control of irrigation canal operations is becoming increasingly obvious due to the less than desirable performance of manually controlled large-scale irrigation systems. In addition, the existing rigid schedules do not allow the farmers to tap the full potential of modern irrigation technology at the farm level. Delivery schedules that are more flexible would allow the farmers the needed flexibility to achieve higher efficiencies at the farm level. The flexible delivery schedules, however, make the manual operation of irrigation canals very difficult. To overcome this difficulty, attempts have been made, in the past, to develop local (Buyalski and Serfozo 1979; Zimbelman 1981; Burt 1982; Chevereau et al 1987) and centralized (Rogier et al 1987) control algorithms for demand delivery operation of irrigation canals. However, the derivation of the control algorithms was based upon a tedious trial and error procedure using extensive simulations of the unsteady open-channel flow model.

Optimal control theory provides a well defined methodology to derive gate control algorithms (Corriga et al 1982; Balogun 1985; Reddy 1990), and eliminates the trial and error procedure that has been used in the past. Balogun (1985) used optimal control theory for deriving a gate control algorithm for a centralized control system. This procedure, however, handled only initial disturbances without accounting for continuously acting external disturbances (changes in lateral withdrawal rates). Reddy (1990) developed a local control algorithm for controlling individual gates in the presence of both initial and external disturbances acting on the system. However, in the derivation of the control algorithm, the interactions between the pools were neglected. Therefore, there is a need for developing a simple control algorithm for operating a series of gates on an irrigation canal. The objective of this paper is to present a decentralized observer-controller algorithm for demand delivery operation of irrigation canals in the presence of external disturbances.

Mathematical Model

In the operation of irrigation canals, decisions regarding gate openings in response to changes in water withdrawal rates into lateral or branch canals is required to maintain the depth of flow or the volume of water in a given pool at the target value. This problem is similar to the process control problem in which the state of the system is maintained close to the desired value by using real-time feedback control. To apply the linear control theory, the Saint-Venant equations of open-channel flow, which are presented below, were linearized:

$$\frac{\partial y}{\partial t} = -\frac{1}{T} \left(\frac{\partial Q}{\partial x} + w \right) \quad (1)$$

$$\frac{\partial Q}{\partial t} = -\left(\frac{\partial \left(\frac{Q^2}{A} \right)}{\partial x} + gA \left(\frac{\partial y}{\partial x} - S_0 + S_f \right) \right) \quad (2)$$

in which y = depth of flow, m; Q = flow rate, m^3/s; T = flow top width, m; A = cross-sectional area of flow, m^2; S_o = canal bed slope; S_f = friction slope = $Q|Q|/K^2$; g = acceleration of gravity, m/s^2; w = lateral outflow (positive) or inflow (negative), $m^3/s/m$; K = hydraulic conveyance of the channel = $AR^{2/3}/n$; n = Manning friction coefficient; R = hydraulic radius, m; t = time, sec; and x = distance, m.

In Eqs. 1 and 2, the spatial derivatives were replaced by finite-difference approximations, by dividing the pool into few segments (N number of nodes). The forward-, the central-, and the backward-difference schemes were applied to the first, the intermediate, and the last nodes, respectively, of each pool. The turnouts can be located any where in the pool, but the location must be specified for modeling purposes. To solve Eqs. 1 and 2, appropriate boundary conditions at the gates need to be specified. These boundary conditions were expressed in terms of the continuity and the gate discharge equations given by:

Continuity Equation:

$$Q_{i-1,N} = Q_{i,1} = Q_{gi} \quad (3)$$

Gate Discharge Equations:

$$Q_{gi} = C_{di}\, b_i\, u_i \sqrt{2g\,(y_{i-1,N} - y_{i,1})} \quad (4)$$

in which C_{di} = gate discharge coefficient; b_i = width of gate i, m; u_i = opening of gate i, m; $y_{i,1}$= depth of flow at the first node of pool i, m; $y_{i-1,N}$ = depth of flow at node N of pool i-1, m; $Q_{i,1}$ = flow rate at the upstream end of pool i, m^3/s; $Q_{i-1,N}$ = flow rate at the downstream end of pool i-1, m^3/s; Q_{gi} = flow rate through upstream gate of pool i, $m^3/s/m$; i = pool index; and j = node index (1 to N). In Eq. 4, the change in bottom elevation of the canal across the gate was assumed negligible.

Lateral Withdrawals

The lateral canals were assumed to be located immediately upstream of the last node in each pool (Figure 1). Though the lateral withdrawal was concentrated at one point, for modeling purposes, it was assumed to be uniformly distributed between the adjacent nodes, and was related to w of Eq. 1 as follows:

where s = Δx in the case of a backward difference scheme, and s = $2\Delta x$ in a central difference scheme; and $q_{i,N}$ = lateral withdrawal rate at node N of pool i, m^3/s. Similar relationships can be

$$w = \frac{q_{i,N}}{s} \tag{5}$$

derived for situations where the laterals are located throughout the length of pool.

Linearization of System Equations

The linearized model was derived based upon an initial steady state condition. Using the Taylor series expansion around the initial point and truncating terms higher than the first-order, a set of linear equations was obtained (Balogun 1985; Dia 1990). The discrete-time version of these equations for a canal reach with M pools is:

$$\mathbf{\delta x}(k+1) = \mathbf{\Phi}\ \mathbf{\delta x}(k) + \mathbf{\Gamma}\ \mathbf{\delta u}(k) + \mathbf{\Psi}\ \mathbf{\delta q}(k) \tag{6}$$

$$\mathbf{\delta y}(k) = H\ \mathbf{\delta x}(k) \tag{7}$$

in which $\mathbf{\delta x}(k)$ = ℓ x 1 state vector; $\mathbf{\delta u}(k)$ = m x 1 control vector; $\mathbf{\phi}$ = ℓ x ℓ system feedback matrix; $\mathbf{\Gamma}$ = ℓ x m control distribution matrix; $\mathbf{\Psi}$ = ℓ x p disturbance distribution matrix; $\mathbf{\delta q}(k)$ = p x 1 matrix representing external disturbances (changes in water withdrawal rates) acting on the system; $\mathbf{\delta y}(k)$ = r x 1 vector of outputs (measured variables); $\mathbf{H}$ = r x ℓ output matrix; ℓ = number of dependent variables in the system; m = number of controls (gates); p = number of lateral canals on the supply canal in the given reach; k = sampling instant; and r = number of measured outputs. In Eqs. 6 and 7, the variables $\mathbf{\delta x}, \mathbf{\delta u},$ and $\mathbf{\delta q}$ are defined as follows:

$$\mathbf{\delta x} = (\delta y_{1,1}, \delta y_{1,2}, \delta Q_{1,2}, \ldots\ldots\ldots, \delta y_{M,N-1}, \delta Q_{M,N-1}, \delta y_{M,N})' \tag{8}$$

$$\mathbf{\delta u} = (\delta u_1, \delta u_2, \ldots\ldots, \delta u_{M+1})' \tag{9}$$

$$\mathbf{\delta q} = (\delta q_{1,1}, \ldots, \delta q_{1,N}, \ldots\ldots\ldots, \delta q_{M,N})' \tag{10}$$

in which $\delta Q_{i,j}$ = variation in flow rate at node j of pool i, m^3/s; $\delta y_{i,j}$ = variation in depth of flow at node j of pool i, m; δu_i = variation in upstream gate opening of pool i, m; δu_{i+1} = variation in downstream gate opening of pool i, m; and $\delta q_{i,j}$ = variation in water withdrawal rate at node j of pool i, $m^3/s/m$. The elements of the matrices Φ, Γ and Ψ depend upon the initial condition.

Design of Controller

Equations 6 and 7 can be used to simulate the system dynamics as a function of time, given the initial conditions ($\delta x(0)$), the external disturbances acting on the system (δq), and the gate opening (δu). However, in canal operations the gate opening is the unknown. To achieve a desired system dynamics, given the initial condition and the disturbances (known or unknown), the selection of an appropriate gate opening becomes a trial and error procedure. The concepts of control theory can be applied to eliminate this trial and error procedure, and derive a direct solution for gate opening. To apply control theory, the matrices **Φ**, **Γ**, and **H** must satisfy the stability, controllability, and observability properties (Kailath 1980). The function of a control algorithm is to bring an initially disturbed system to the desired target value in the presence of external disturbances acting on the system. For this problem, a proportional plus integral (PI) control (Figure 1) of the following form was derived using optimal control theory:

$$\boldsymbol{\delta u}(k) = -\boldsymbol{K}\ \boldsymbol{\delta x}(k) \tag{11}$$

where **K** = controller gain matrix. A large value of **K** requires a high energy input, and might cause overtopping of canal banks. Conversely, if the control is not of sufficient magnitude, the system would return to the target value very slowly, causing large deviations in depths of flow or volume of water stored in the pools. Therefore, there is a trade-off between the rate of return to the equilibrium condition and the overtopping of canal banks. The elements of the controller gain matrix **K** are obtained by solving the algebraic matrix Riccati equation, which is a celebrated equation in control theory, and a variety of techniques are available to solve this equation (Kailath 1980). In Eq. 11, once the elements of **K** matrix are available, either measured or estimated values of $\boldsymbol{\delta x}(k)$ are used to calculate the desired variation in the opening of gates. (Figure 1).

In a centralized control scheme, the control algorithm of each gate (Eq. 11) needs real-time information (flow depths and flow rates at the nodal points) from all the pools in the system. This increases the computational complexity and the amount of data transmission to the central control station. Conversely, in a decentralized control scheme, the control algorithm of each gate requires real-time information from only a few adjacent pools. The decentralized control algorithm presented in this paper (Figure 2) needs information only from the three adjacent pools (one upstream and two downstream pools). This minimizes the amount of data transfer between the pools and the control station(s).

Design of Observer

In the implementation of the control law defined by Eq. 11, measured values for all the variables- depth and flow rate at all the intermediate nodes of each pool, depth at the first and the last nodes of each pool- must be available for feedback from only the adjacent pools in the case of a decentralized control scheme (Figure 3). This is undesirable from an economic point of view. Since it is very expensive to measure all the state variables, particularly the flow rates, values for some of the state variables must be reconstructed from the available data collected from an irrigation canal. An 'estimator' or 'observer' can be used to minimize the number of measured state variables. An observer is a mathematical model of the given system which predicts values for the state variables that are not measured, based upon measured values of a few state variables (upstream and downstream depths of flow in each pool). Reddy et al (1992) presented a technique for designing an observer for local control systems. The same technique can be extended for the decentralized control scheme presented in this paper. The observer (or estimator) equation is given as follows:

$$\boldsymbol{\delta}\hat{\mathbf{x}}(k+1) = \boldsymbol{\Phi}\ \boldsymbol{\delta}\hat{x}(k) + \boldsymbol{\Gamma}\ \boldsymbol{\delta u}(k) + \boldsymbol{L}(\boldsymbol{\delta y}(k) - \boldsymbol{H}\ \boldsymbol{\delta}\hat{x}(k)) \tag{12}$$

in which **L** = observer gain matrix. In Eq. 12, the estimated values for the state variables are driven by the difference between the measured and estimated values for the selected state variables. Here, it is assumed that only the upstream and downstream depths of flow in each pool are measured. The dimension of the **L** matrix is equal to the number of state variables estimated including the integral control variables, and the number of measured state variables. Since the observer resides in the computer, in order to keep the on-line computation to a minimum, and to minimize the cost of data communication, a decentralized observer is preferred compared to a global observer. The decentralized observer has a block-diagonal structure. A schematic of a feedback control system with an observer in the loop is presented in Figure 2.

The elements of the observer gain matrix are selected such that the estimated values of the state variables approach the actual values as quickly as possible. This can be done either by using a pole (eigenvalue) placement technique or by designing a Kalman Filter. In this paper, the pole placement technique is used to design the observer. In the presence of random disturbances, assuming that a model is available to simulate the disturbances, the Kalman Filter is an appropriate choice. The application of Kalman Filter (adaptive type) will be discussed in a subsequent paper.

Results and Discussion

To demonstrate the applicability of the technique discussed above, an example problem was considered. The data presented in Table 1 were used to derive the elements of Φ , Γ and Ψ matrices, and the initial steady state gate openings (Table 1) for all the 6 gates. The opening of the 6th gate was then fixed at its initial steady state value. The discrete-time matrices were used in deriving the elements of the controller gain matrices for a decentralized control scheme. The performance of the control algorithms were evaluated by introducing known disturbances into the pools (Table 1). In the evaluation, an approximate estimation of the variation in the volume of water stored in the pools was obtained by the following expression:

$$\delta V = \frac{A_{sss}}{4}\left[0.50\,\delta y_1 + \delta y_2 + \delta y_3 + \delta y_4 + 0.50\,\delta y_5\right]$$

in which A_{sss} = steady state top water surface area (m^2), δV = variation in the volume of water stored in any given pool (m^3); and δy_i = variation in the depth of flow at node i (i = 1 to 5), in a given pool. The values of A_{sss} and the initial volumes of water stored in the pools are presented in Table 2.

Table 1. Data used in the simulation study

Parameter	Pool/gate 1	Pool/Gate 2	Pool/Gate 3	Pool/Gate 4	Pool/Gate 5
Pool:					
length(m)	7000	7000	7000	7000	7000
width (m)	12.25	12.25	12.25	12.25	12.25
bottom slope	0.0001	0.0001	0.0001	0.0001	0.0001
side slope	1.50	1.50	1.50	1.50	1.50
initial lateral flow rate (m3/s)	9.00	6.00	5.00	6.00	5.00
initial downstream depth (m)	4.00	3.40	2.60	2.05	1.66
downstream flow requirements (m3/s)	36.00	30.00	25.00	19.00	14.00
Gate:					
width(m)	18.25	18.25	18.25	18.25	18.25
discharge coeff.	0.83	0.83	0.83	0.83	0.83
initial opening(m)	0.32	0.58	0.45	0.48	0.44
Disturbances(m^3/s)	5.00	6.00	4.00	3.00	3.00

Upstream reservoir elevation (m)	110.10
Downstream reservoir elevation (m)	99.90
Upstream invert elevation (m)	102.10
Downstream invert elevation (m)	98.60

Table 2. Initial Top Water Surface Area and Volume of Water Stored in Pools

Pool #	Average Top Width (m)	Pool Length (m)	Surface Area (m^2)	Initial Volume (m^3)
1	23.85	7,000	166,950	488,110
2	22.07	7,000	154,497	392,700
3	19.79	7,000	138,548	281,120
4	18.29	7,000	128,044	214,620
5	17.20	7,000	120,365	169,610

Disturbances in the form of increased flow rates into all the 5 laterals in the system were introduced (Table 1), and the results of the simulation study are presented in Figure 3. The maximum volume variation of 5400 m^3 occurred in pool 2, followed by a variation of 4800 m^3 in pool 3 (Figure 3a). These resulted in 1.38 % and 1.71% of the initial volumes of water stored in pools 2 and 3, respectively, which are not significant variations. After an hour of introduction of the disturbances, the variations in the volume of water stored in the pools bounced back to the initial volumes, and gradually became positive. However, the positive variations in the volume of water stored were smaller than the initial negative variations. These variations in the volume of water stored in the pools were considered to be acceptable.

The variations in gate opening in response to the simultaneous disturbances were also simulated. The maximum steady state variation in the opening was 0.31 m for gate 2 followed by a variation of 0.20 m for gate 4. The variation in the opening of all the other gates was in the range of 0.15 m to 0.20 m (Figure 3b). These variations in gate opening were considered reasonable.

In the above simulations, it was assumed that all the disturbances were positive (increased flow rate into laterals) and started at the same time. Under field conditions, all the disturbances may not be positive and start at the same time. The performance of the system in terms of maintaining a constant volume in the pools would increase when the disturbances acting on the system are temporally distributed, i.e. do not start at the same time. In addition, the performance of the system would be even better in the presence of spatially distributed negative disturbances (either a decreased flow rate into a lateral or a distributed source of inflow into the canal, for example groundwater pumped into the canal) acting on the system.

Figure 4 presents the results obtained using a decentralized estimator in the feedback loop. It is obvious from the Figure that the gate openings calculated using the estimated values approach the calculated gate openings using the measured values of the state variables within 1 hour of the simulation period. This is an acceptable performance of the combined observer-controller algorithm. However, before implementing the algorithm in the field, the performance of the control algorithm must be evaluated using the nonlinear, unsteady open-channel flow model. This will be done in the near future.

An ideal control algorithm is one that would yield a zero percent deviation in the volume of water stored in the pools in the presence of external disturbances acting on the system. But this would result in larger rates of water transfers between the pools, and larger deviations in the depths of flow. In order to minimize the deviations in the depths of flow, some variation in the volume of water stored in the pools must be accepted. The control algorithm presented here strikes a balance between the deviations in the depth of flow and the deviations in the volume of water stored in the pools. In all the simulations, the depth at the middle node of the pools remained more or less constant, and the water surface elevation always pivoted about that point, which is similar to the BIVAL control mechanism (Chevereau et al 1987). The maximum variation in the depths of flow was at the first and the fifth node in each pool. In this particular example problem, the laterals were located at the downstream end of the pools. Because of the deviations in the depth of flow, the turnouts at the upstream and downstream ends of the pools, if any, must be fitted with discharge regulators to deliver the required flow rate into the laterals under variable head in the supply canal.

Summary and Conclusion

Using spatial discretization and the Taylor series expansion, a linear lumped parameter model of open-channel flow was obtained. The canal control problem was formulated as an optimal control problem, and the set of linear equations obtained were used to derive a decentralized control and estimation scheme for operation of a series of irrigation canal gates. For the decentralized control algorithm presented in this paper, the control algorithm of each gate needs information only from three adjacent pools at most. A proportional plus integral control was derived to handle both the initial and the external disturbances acting on the system. A local, decentralized observer was designed to reconstruct most of the state variables of the system, given the measured values of only the upstream and downstream depths in each pool.

To test the performance of the control algorithms, a canal reach with 5 pools was considered. The performance of the control algorithm was evaluated by simulating the dynamics of the system in the presence of several external disturbances in all the 5 pools. The maximum variation in the volume of water stored in the pools was less than 1.6 % when the total magnitude of the disturbances acting on the canal system was 21 m^3/s. This represented an increase in flow rate of close to 50% of the initial flow rate into the canal. The performance of the decentralized control scheme in terms of maintaining a constant-volume of water in the canal pools was found to be acceptable. Though only 5 pools were used in this paper for evaluating the performance of the technique, the same methodology can be used to derive control algorithms for irrigation canals that have large (more than 5) number of pools (or gates).

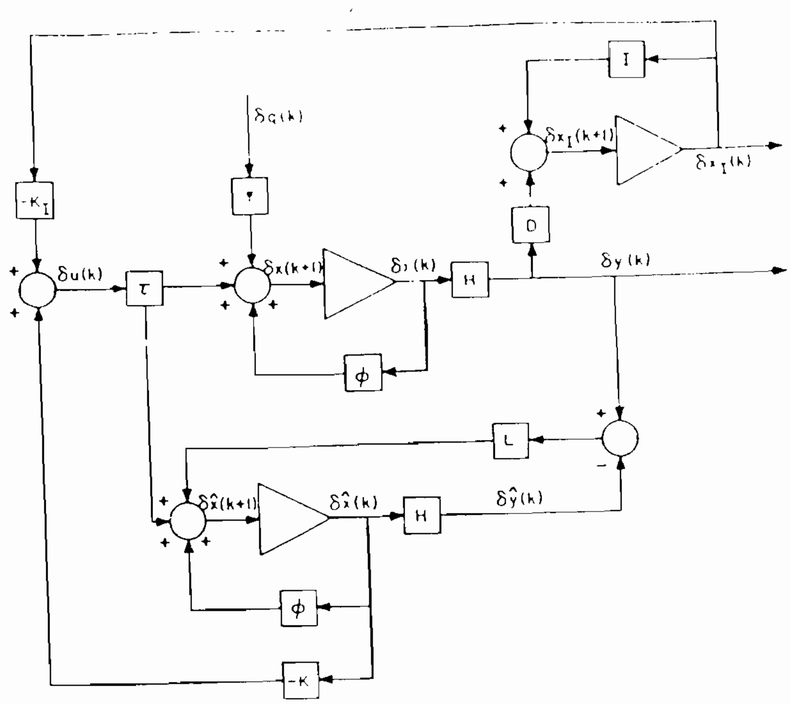


Figure 1. Schematic of a State Feedback Control System

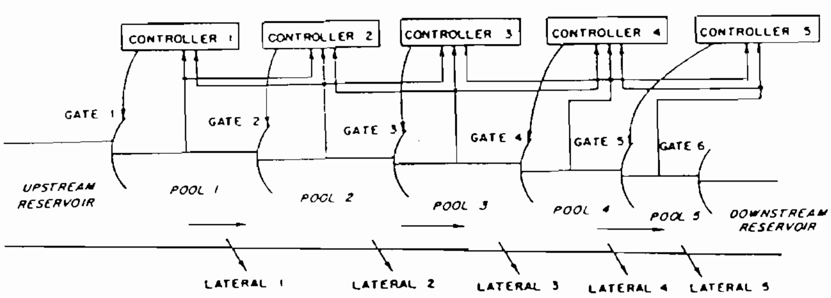
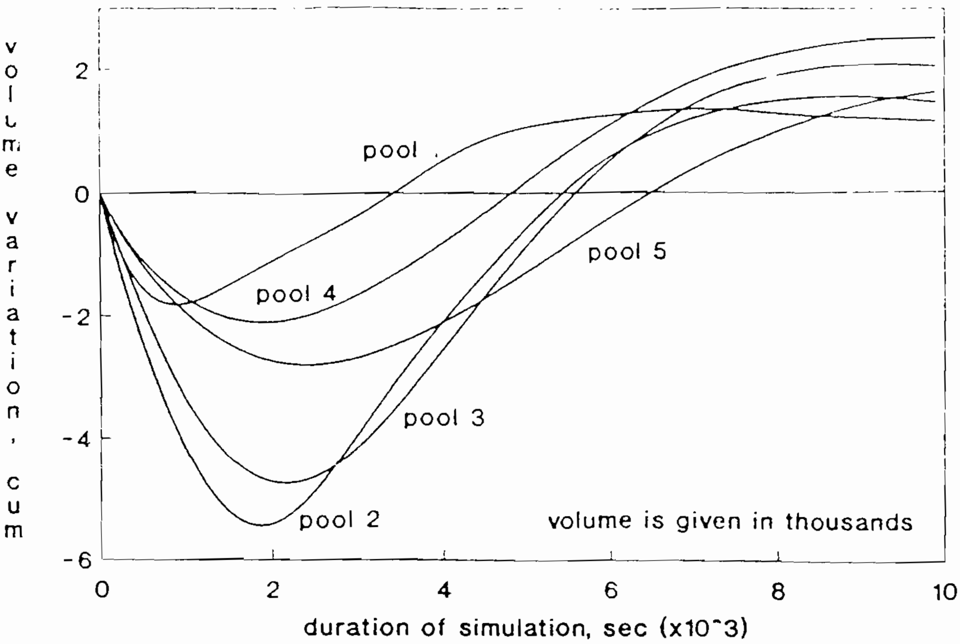
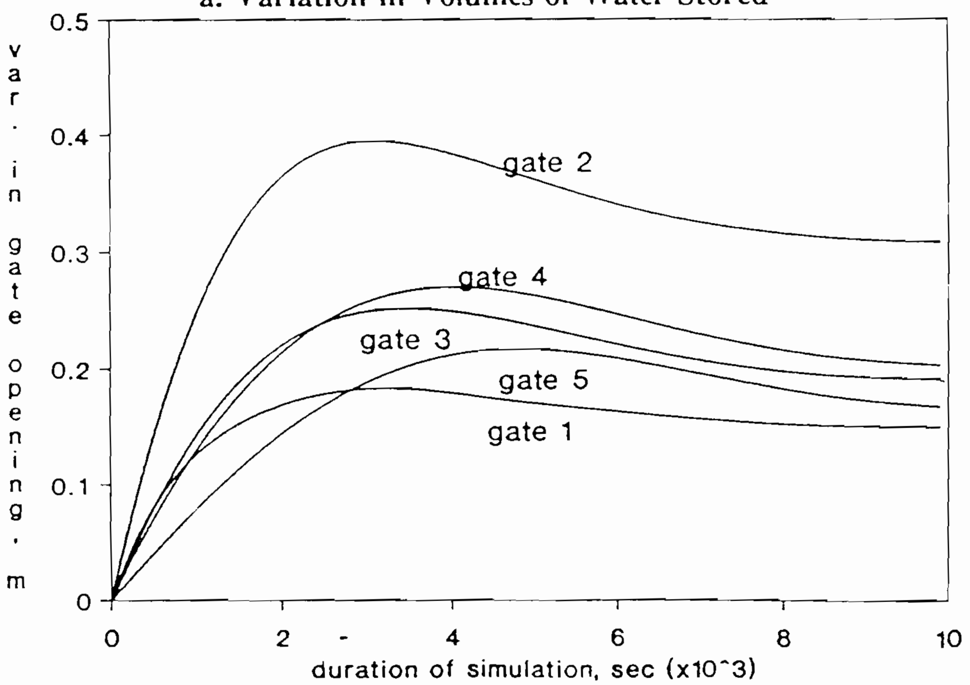


Figure 2. A Decentralized Control Scheme



a. Variation in Volumes of Water Stored



b. Variation in Gate Opening

Figure 3. Canal Response to Disturbances in all Pools

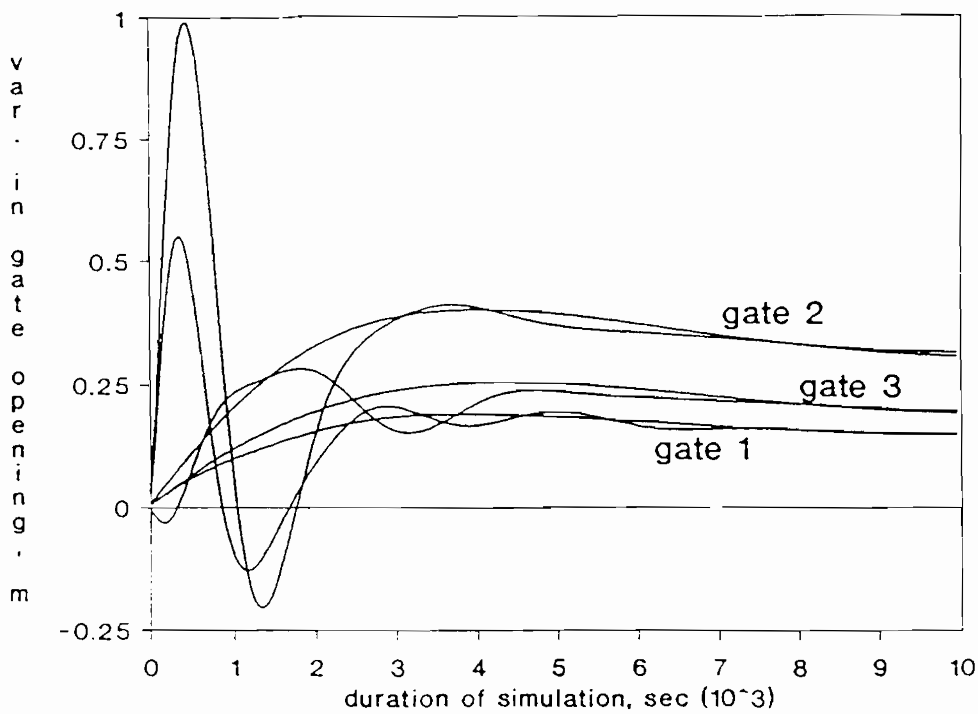


Figure 4. Variation in Gate Opening Using the Observer

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