Resource value, productivity and ecosystem integrity: an intertemporal water resource management tool in the a river basin

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Abstract

Sustainability constraints are quickly catching up with economic developments in water catchment areas. Inefficient water use by households, primary producers and industrial sectors may account for a large proportion of depleted water in most African river basins. Existing integrated water allocation tools are seldom comprehensive enough to account for all possible uses of water in a river basin. This paper attempts to model water resource allocation in a river basin by specifically posing sustainability constraints in an integrated model. The results show that in "open access" property regimes, resource users are myopic and could deplete water and vegetal cover. In a "common property" regime however, individuals are less callous and would be willing to pay a premium, ex-ante and ex-post, for the maintenance of the resource in a manner that maximizes private and collective welfare. In particular, when all resource users face some ecological restrictions on the use of water and complementary resources, sustainability could be achieved without reducing the flow of private and social benefits to all resource users collectively.

Keywords
Integrated water resource management
Ecosystem integrity
Maximum principles
Sustainability constraints
Open access
Common property

Introduction

Sustainability constraints are quickly catching up with economic developments in water catchment areas. Inefficient water use by households, primary producers and industrial sectors may account for a large proportion of depleted water in most African river basins. Resource managers often bear the resultant blunt of criticism, which is reasonable considering the enormous institutional failures to acknowledge multiple uses of water and at the same time manage conflicts among users.

Contemporary resource management regimes represent a movement away from centralized decision making to a form of participatory approaches that seek the inputs of all agents that will be affected by the policy. The rationale for inclusion rather than exclusion of the public in decision-making is that there are a variety of actors directly or indirectly affected by resource development. The actors exert influence on the resource and on others depending on their ecological importance, power (e.g. extent of their operations, market power, negotiation skills, customary rights, statutory authority, etc), their objective (value) functions, and the level of, and the system of cooperation among the actors.

When resource managers fail to understand the system even in its simplest and uncooperative form, imposition of a policy will have ex-post results that could significantly divert from sustainability paths. Inclusion on the other hand, allows the interaction among actors to
influence policy ex-ante, and leave the ex-post assessment of the effectiveness of policy to focus on improving the foundation of the policy, which is rooted in the structure of cooperation among actors.

Ex-ante, integrated water allocation tools are seldom comprehensive enough to account for all possible uses of water in a river basin. In most cases only a subset of users, predominantly irrigators, households, and pastoralists are given prominence understandably because they exhibit intense and at times volatile competition for water. Exclusion of other users, especially embedded species in the ecosystem, is less acknowledged than the empirical usefulness of the decision models themselves.

While economic analyses are the easiest way of conceptualising resource allocation, a lot can be gained from broadening the scope to include institutional and ecological dynamics. Economic analyses emphasize on measuring efficiency of water (utilization) as the benefit realised from an activity as a function of the amount of water used directly by the activity. The Pareto concept of efficiency in allocation requires that water be allocated to the highest bidder, or the one whose output per unit of water used is the highest. Unfortunately, efficiency does not guarantee equity and sustainability in resource allocation. Sustainability considerations require a prototype model formulated along the lines of the Walrasian General Equilibrium model combining the value functions of all the agents in the river basin, including the resource requirements of the ecologically embedded agents.1

This paper attempts to model water resource allocation in a river basin by specifically posing sustainability constraints in an integrated model.2 The primary hypothesis that is tested analytically is whether the water resource will be depleted in the absence of sustainability rules. The secondary hypothesis is whether individual agents may find it in their best interest to cooperate and share some costs in maintaining the resource. Last but not least, the paper attempts to predict the time path of the resource assuming different initial scenarios, different strategies by the actors and a variety of policy directions.

The model

A river basin is a natural unit for integrated water resources planning and management, since water interacts with and to a large degree controls the extent of other natural components such as soil, vegetation, and wildlife (Cai, et al., 2003). Water supplies in a catchment area come from surface sources (including the rivers, streams, and marshes), and from ground sources (aquifers). Generally, water is considered a renewable resource, although some sources, especially aquifers could be depleted. However, if the rate of groundwater extraction is less than or equal to the rate of recharge, water use from an aquifer could be sustained indefinitely (Conrad, 1999).

Economic models could help in the decision-making processes dealing with complex systems. Important economic issues in integrated economic-hydrologic river basin modeling include transaction costs, agricultural productivity effects of allocation mechanisms, intersectoral water allocation, environmental impacts of allocations, and property rights in water for different allocation mechanisms. Some of these models attempt to determine the optimal spatial and temporal allocation of a complex water resource system and to examine the relative performance of various policies (Rosé grant, et al., 1996; Cai, et al., 2003).

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1 For exposition purposes, the terms “catchment area” and “river basin” are used interchangeably although a catchment area is more encompassing and extensive and may consist several river basins. 4
2 The Brundtland Commission defined Sustainable Development as “development that meets the needs of the present generation without compromising the ability of future generations to meet their own” (The World Commission on Environment and Development (WCED), 1987).
In most communities, surface and groundwater are an open access resource with no exclusive property assigned to specific individuals. If however the community elects to collectively manage the resource, the water becomes a common property for which each voting member of the community is responsible, both for use and maintenance. Integrated catchment management models assume either an open access or a common property and simulate interactions among the water users on one hand, and between the government or public agencies (NGOs) investing in the community public program and the community on the other hand.

In this study, the water from the various surface and ground sources are a vital part of a resource ecosystem. The ecosystem consists of commercial farmers (mainly irrigators), smallholder farmers, household users, pastoralists, public agencies, and animal and plant species embedded in the system. The model will primarily describe the resource base in terms of the quantity of water in the catchment area. The primary control variable is therefore the amount of water extracted per unit of activity in each sector. Other riparian resources are of a secondary consequence since they may not exist if the primary resource falls below a certain level.

**Commercial farmers**

Commercial farmers are interested in land and water for productive use. They produce crops for local consumption and for trade. They control the use of the water resource by having licensed activities, and by having permanent tenure for their landholdings. Commercial farmers are typical profit *maximizers* who choose the quantities of inputs employed subject to production technology.

\[ \eta_i (L,K,S,W) = P(q)q - C_i (L,K,S,W) \]  \( \text{(1)} \)

Where \( \eta \) is farmer \( i \)'s profit, expressed as a function of market price \( P(q) \) for her produce, \( q \) and costs, \( C_i \) which are also a function of labour (L), capital (K), land (S) and water (W).

**Households and smallholder farmers**

Smallholder farmers are also interested in productive use of water and land. Unlike commercial farmers, most of the crops produced by smallholder farmers are grown on tenable land and are mainly used for subsistence consumption. In terms of commercial interests, smallholder farmers are usually not fully integrated in the inputs and produce markets and so their assumed market behaviour is similar to those of subsistence households. It therefore becomes almost impossible to separate the interests of smallholder farmers from household water users since they have almost similar characteristics.

Household water users are more concerned with amenities that water provides for household production functions such as cooking, washing, bathing, and drinking. Households also value the harvest of fish, weeds, reeds, and other materials along watercourses or marshes. Together, households and smallholder farmers are a formidable resource extracting 'union' that may threaten the existence of commercial resource users.

Households (and smallholder farmers) are assumed to maximize a utility function, which takes as its arguments, the subsistence production, leisure, and amenities from water.

\[ U_j (L,T,W) \]  \( \text{(2)} \)

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3 The description of stakeholders in a river basin is consistent with characteristics observed in the Malawi formal water catchment management areas. Similar patterns could be observed in the SADC region, especially in Mozambique, Tanzania, Zambia and Zimbabwe.
Where $U_j$ is household $j$'s utility as a function of subsistence level of consumption, $I$, the maximum available time for leisure and labour, $T$, and $W$ is the household's consumption of water. For internal consistency a household is assumed to have maximum subsistence consumption, $I_{\text{max}}$, and a limit, $L_{\text{max}}$, on the capacity to supply labour input. Consequently household excess production (or income) and leisure hours are respectively defined as below.

$$M_j = I_j - I_{\text{max}} \quad \ldots (3)$$

$$Z_j = T_j - L_{\text{max}} \quad \ldots (4)$$

**Pastoralists and nomadic grazers**

Pastoralists are interested in water and patches of vegetation along river banks, and natural vegetation and forests elsewhere in the catchment area. They produce animal and diary products for local consumption and for trade. They are typically nomadic, moving over large areas and sometimes, encroaching and trampling over land, which belongs to formal interests. Pastoralists claim customary rights over water and pasture, and the rights are usually passed on from generation to generation.

Grazing pressure may lead to discontinuous and sometimes irreversible change in the state of the grassland. The degradation of the grassland is marked by changes in the density and distribution of shrubs and perennial grasses, as well as in the patchiness of the landscape (Perrings, et al., 1997). The main concern for pastoralists then is to maximize productivity of their herd, subject to the evolution of vegetation and water. Formally, they maximize profits from livestock subject to the availability of palatable grasses and water.

$$\text{Maximize } NPB_t = p_h - c(x_t, v_t, w_t) \quad \ldots (5)$$

Subject to herd and resource dynamics

$$x_{t+1} = x_t + f(x_t, v_t, w_t) - h_t \quad \ldots (6)$$

$$v_{t+1} = v_t + g(x_t, v_t, w_t) \quad \ldots (7)$$

$$w_{t+1} = w_t + r(x_t, v_t, w_t, \varepsilon_t) \quad \ldots (8)$$

Where at time $t$, $NPB_t$ is the net benefit to a pastoralist from animal and diary products, $x_t$ is the biomass or population of the herd, $h_t$ is the product (meat, diary, hides, etc) from the herd, $v_t$ is the biomass of vegetal cover, $w_t$ is the amount of water in the catchment area, $\varepsilon_t$ are weather parameters, while $f$, $g$ and $r$ are functions.

**The public sector**

The government or public agencies are included as external stakeholders who may take responsibility for the conservation/preservation of water resources by ensuring equitable distribution of water, and taking on the role of conflict arbitration. Conflicts may arise between a commercial farmer and a pastoralist over the latter’s indiscretion in moving their animals over land and water and the commercial farmer’s property rights over land and water. Households and commercial interests may argue over access and distribution of land and water, while pastoralists may find favour from households as long as animals are kept away from sources of drinking water.
Given the complexity of incentives of the agents using the water resource, the government has three alternative interventions. The first alternative is to do nothing and let agents maximize their own private benefits without worrying about the effects that their actions may have on other users. This is the case of “laissez-faire, open access” resource. Alternatively, the government may wish to impose rules by directly dictating the behaviour of all the agents using the resource. This second case is typical of a “command economy” where the government is not only a producer of public goods and services, but also the owner of means of production, and sole distributor of resources. Lastly, the government may allow some cooperation among users as they agree on how best to collectively manage the water and the natural resources in the catchment area. The latter case is that of “common property” resource.

The next section considers “open access” and “common property” regimes in detail. A “refutable” assumption is that agents maintain their objective functions in both scenarios.

### Analytical Results

Assuming a laissez-faire open access scenario, agents maximize short run benefits since they do not concern themselves with the effects that their actions may have on others. This is in contrast with situations where agents must account for externalities by internalizing both costs and benefits from a series of feedback impacts from their activities. Since feedback effects do not enter the decision functions of the agents, static optima sufficiently describe equilibrium conditions in “open access” allocation problems. The static equilibrium for each agent is found by maximizing the relevant objective function subject to its corresponding constraints.

#### Static optimum in open access

**Commercial farmers**

Commercial farmers maximize profits by selecting inputs at the point where the real marginal contribution of the input to output is exactly offset by the real cost of the input.

\[
\frac{\partial \eta}{\partial L} = \frac{\partial \eta}{\partial K} = \frac{\partial \eta}{\partial S} = \frac{\partial \eta}{\partial W} = 0
\]

Where, \( L, K, S \) and \( W \) are as defined above.

Thus it would not be worthwhile for farmers to employ an input whose real addition to revenue is less than what it costs to retain the input. As shown in Appendix 1 (Equations A1.1- A1.4), this implies that farmers employ inputs only when the value marginal product of the input is greater or equal to the marginal cost of the input. Further, the first order conditions imply that, for commercial farmers, it is optimal to use the largest amount of water up to the point where the marginal benefit of water is zero. This is especially true when the costs of irrigation equipment and operations are internalized but not charged to the amount of water pumped from aquifers or rivers. Thus, for water resources, which are freely available, the optimal solution is to use as much water as is possible to drive the value marginal product of water to zero. As a consequence, water from aquifers and rivers is used inefficiently and could be depleted.

**Households and smallholder farmers**

Smallholder farmers maximize utility from subsistence consumption (of water and agricultural output) and from leisure where, assuming a quasi-linear technology with non-negativity constraints (i.e, \( I > 0, T > 0, \) and \( W > 0 \)):

\[
U(I, T, W) = \xi(I - I_{max})^\gamma + \tau(L_{max} - T)^\nu + \xi W^n
\]

... (10)
Substituting terms from definitions (3) and (4) above, the relevant total differential is as derived below.

\[ dU = t_1 t_2 (M)^{a-1} dI - t_2 t_3 (Z)^{b-1} dT + t_3 t_4 W^{c-1} dW \]  

... (11)

Utility is maximized when the shadow wage rate from the uncompensated time spent producing excess food is equal to the shadow price of time spent collecting water (Appendix 1, A1.5-A1.10). Although water is available for free in the rivers and aquifers, it has an opportunity cost equal to the time value of labor in the production of excess food, which could be sold or reserved for future subsistence consumption. Similarly leisure has an opportunity cost which could be measured in terms of forgone subsistence output.

**Pastoralists**

Pastoralists employ labor and procure basic services for maintaining a given herd. They also face costs of time spent searching suitable pasture and water for their animals. Assuming labor and procurements are a fixed proportion of the herd population, the pastoralists maximize the net private benefits by choosing a stock level and inadvertently, the level of water in the rivers and aquifers, and the form of vegetation cover.

\[ \frac{\partial NPB}{\partial x} = \frac{\partial NPB}{\partial v} = \frac{\partial NPB}{\partial w} = 0 \]  

... (12)

These conditions, derived fully in Appendix 1 imply that pastoralists would be maximizing net benefits by equating the value marginal private benefits from animal products (offtake) to the cost of maintaining an additional unit of herd. The marginal net private benefits and the cost of the herd change with respect to changes in the vegetal cover around the river basin, in addition to the level of water. In particular, pastoralists maximize private benefits by equating the change in the value of offtake due to a change in either vegetation or water to the corresponding marginal cost of water or vegetation. Just like commercial farmers, pastoralists face zero marginal cost of both river water and vegetation. The static solution therefore implies that pastoralists could use water and vegetation up to the point where the marginal private benefit from offtake is also zero.

Apart from the obvious threat to the sustainability of water and vegetal resources, the nomadic activities of pastoralists pose external costs to the environment and on other resource users. The externalities are usually a crucible for conflicts with other resource users. Unfortunately the absence of a regulator in "open access" resources suggests that conflicts could escalate unless intermediations were installed. Soil degradation is rapid in areas frequented by trampling animals. The change in vegetal cover is often the first noticeable environmental damage but it may take a while before the change in vegetal cover has any impact on water cycle and on low flow in the rivers.

**Dynamic solution**

A cooperative game with decentralized regulation

Water is the main resource of concern for all agents, while as shown above, vegetation cover directly affects the value of offtake and indirectly the level of water in the catchment and the quality of soils in the arable lands. The idea is that by cooperating in the use of water and vegetation, all agents would be able to maximize their private benefits but only if other agents' objective functions attain minimum acceptable values. Hence, within a voting community with universal suffrage, the individuals' objective function is constrained by values chosen by fellow voters, so that where individuals contribute to the maintenance of the public goods, individuals
would care about the magnitude of their own contributions only insofar as these contributions affect the aggregate level of benefits (Bemheim, 1986).

Assuming that a cooperating structure is agreed upon by all the agents in the catchment area, i.e., commercial farmers, smallholder farmers, households and pastoralists set cooperating parameters in the use of water and vegetation, then an inter-temporal value function is the relevant allocation tool. In such a cooperative model, the agents play a series of games from which weights are assigned to their benefits so as to maximize their combined benefits from water and vegetation.

A decentralized public authority may control the way in which the cooperative game is played by either actively playing the role of guardian for the ecosystem in general, or maintaining an environment for fair play among the agents. The decentralized regulator therefore maximizes the aggregate benefits from the activities of all the agents subject to the dynamics of the affected resources. For this particular system the problem is specified below.

Maximize
\[ \sum_{i=0}^{\infty} \rho^i \left( \gamma_1 U_1(I_t, T_t, W_t) + \gamma_2 U_2(I_t, T_t, W_t) + \gamma_3 NPB \right) = \sum_{i=0}^{\infty} \rho^i GV(.) \] ... (13).

Subject to equations 6, 7 and 8

Where \( \eta(.) \), \( U(.) \), and \( NPB \) are as defined above, \( \rho \) is the discount factor at time \( t \), \( \gamma_i \), \( i = 1, 2, 3 \) are weights, and \( \{ i = 0 \} \) is the current period.

The aggregate benefits are cast in a discrete form in order to capture seasonal changes in water level and vegetal cover. The Lagrangian for the cooperative model is specified as follows.

\[ L = \sum_{i=0}^{\infty} \rho^i \left( GV(.) + \rho \lambda_{i+1} \left[ x_{i+1} + f(x_{i+1}, v_{i+1}) - h_{i+1} - c_{i+1} \right] + \rho \theta_{i+1} \left[ v_i + g(x_i, v_i, w_i) - v_{i+1} \right] \right) \]

\[ \rho \psi_{i+1} \left[ w_i + r(x_i, v_i, w_i, z_i) - w_{i+1} \right] \] ... (14).

The maximum principles for this problem are derived in Appendix 2.

Implications of the maximum principles

Equation A.2,4 says that the marginal benefit from any future additions to the animal stock of the pastoralists should equal the weighted marginal benefit from offtake in the current period, plus the future benefit from maintaining the current stock and any small additions to it, plus the future cost that the herd would impose on the environment as measured by the change in vegetation and water level. The obvious implication from A.2,4 is that if the vegetation or the water level falls as a result of the increase in stock level, the future net benefit from offtake falls by the corresponding shadow value of the exhausted resource. There could also be benefits if it were possible to control the availability of water or the rate of growth of vegetation by changing the herd stocking level.

There are other externalities associated with grazing practices especially of nomadic livestock. These include the imposed cost on farmers who own less or no animals. Soil compaction due to perennial trampling necessitates frequent tilling of irrigation land for water to infiltrate during the growing season. When the first rains come, a significant amount of fertile soil is lost through erosion since the frequent tilling leaves the topsoil unstable. Soil maintenance costs (to regain
fertility and structure) are therefore not negligible. Although the nutrient requirements of grass in the grazing lands is lower compared to that of crops, the destruction to the soil structure could leave the grazing land less productive in terms of consumable vegetation output per hectare.

The conventional approaches to rangeland management (equilibrium systems or range succession models) use livestock numbers as a tool for controlling rangeland degradation. Unlike a free-range animal grazing system typical of wildlife, livestock do not adjust their densities in relation to grassland productivity. This is where the aggregate stocking rate needs to be checked by the regulator, vis-à-vis the current carrying capacity of the grazing land. Nomadic grazers (wild animals), on the other hand, would move seasonally in response to grassland productivity, structure and species composition (McNaughton, 1985).

Equation (A.2.5) states that the value of an additional unit of vegetation, in situ, in period \( t \) (also called the user cost of vegetation) should equal the weighted marginal benefit to pastoralists in the current period measured by the change in the net benefits from offtake and the future net growth in the stock with respect to the change in vegetation. When vegetation is optimally managed, the marginal benefit from standing vegetation includes the user cost of vegetation, i.e., the value that an unharvested unit would convey in the next period, \( \rho \theta_{i,t} \left[ 1 + \frac{\partial r}{\partial v_i} \right] \), and the impact that the biomass of vegetation will have on water level and on recharge in the next period, \( \rho \psi_{i,t} \left[ \frac{\partial w_i \psi}{\partial v_i} + \frac{\partial r}{\partial w_i} \right] \).

Equation (A2.6) states that when water is optimally managed, the value of an additional unit of water in situ, either in aquifers or in rivers within the river basin will equal the weighted sum of benefits from water accruing to commercial farmers, smallholder farmers, households and pastoralists, plus the marginal benefit that a preserved unit of water will convey in the next period, \( \rho \psi_{i,t} \left[ 1 + \frac{\partial r}{\partial w_i} \right] \), plus the change in water level with respect to net animal stock and net vegetal cover, respectively.

The results above take into account the weights of benefits accruing to all the agents in the catchment and also the extensiveness of actions of specific agents, particularly the pastoralists and irrigators. The implication of the results is that land use options have direct impacts on the time path of water in the river basin. It is a well established hypothesis that changes in vegetal cover may have marked effects on catchment hydrology (O'Shaughnessy, et al, 1983; Smith, et al; 1992). Changes in vegetal cover are brought on by the grazing patterns of pastoralists and the land use alternatives available to smallholder farmers and to commercial farmers. Both afforestation and land clearing could lead to either reduced or improved seasonal low flow depending on the vegetal species involved. The latter suggests that the regulator may be advised to consider restricting species introduced in the catchment area, and also take steps to harmonize land use practices in her jurisdiction.

For instance, in mosquito-infested regions or where productive land is in short supply, afforestation could be used to speed up the process of transforming marshes and springs into inhabitable land. According to Smith, et al, 1992, eucalypts have the greatest impact on low flow than pines because eucalypts show a faster growth rate in the first eight years after planting. If pines and eucalypts were planted in areas where water is seasonally scarce, the impact could be disastrous for agents living downstream.
Conclusion

The paper set out to model the allocation of water among competing users in a river basin, and in particular, attempted to answer the question whether or not sustainable water resource use were possible under alternative property regimes.

The results showed that under “open access” without an external regulator, agents are myopic and would only be concerned with maximizing short run benefits without internalizing externalities imposed on others, and on the ecosystem. As a consequence, water and vegetal cover could be depleted in “open access”. On the other hand, a “cooperative” management regime with a decentralized regulator would be concerned with the feedback impacts of the actions of the agents on each other and on the environment. By setting sustainability rules for both water and vegetation, the regulator maximizes long run benefits for all the agents while internalizing externalities when, and as they occur. A “cooperative” would therefore be concerned with sustaining benefit flows to the community, and hence maintenance of the resource base.

Pastoralists would be expected to follow restricted grazing practices either through seasonal moratoriums on grazing or adopting fodder and stall-feeding technologies. Ecological considerations imply that species of the vegetation will regenerate during restriction. As more of these other sources of feed become significant, other technologies such as stall-feeding could become viable options. This is not to say that stall-feeding or these other technologies would replace the essential role that rangelands play. In particular, the developed technologies would complement grazing lands, and more importantly arrest water and soil degradation.

Alternatively, the grazing land condition can be modified continuously by adjusting stocking rate to a level that either maximizes long-term production per area, or long-term income, or maintains the production per head or the graze land condition at some acceptable level (Westoby, et. al., 1989). Management will therefore be working on variables that are controllable, i.e., the stocking rate per household or village, or at any acceptable stratification of the community. If land use of the rangelands were left unregulated, the intensity of soil and water degradation could reach suicidal proportions.

Pre-commitment and post-commitment (financial and other guarantees entered into by individual members of the community) also influence the effectiveness of institutions. Pre-commitment refers to the financial and other contributions towards the management of the common property resource. The pre-commitment cost should not be such as to exclude others from accessing the resource, but at the same time should be of meaningful value. Theoretically, the values pre-committed should equal the marginal benefits accruing to the respective participants. The post-commitment refers to the system of punishment for violating use restrictions. The idea behind post-commitment guarantees is to raise the cost of environmental shirking. Post-commitment works if the regulating institution can observe violators (monitor behavior) and apply penalties.

Net irrigation could be used as a measure of water utilization by irrigated agricultural crops. The major focus for the regulator would be minimizing inefficiencies in distribution of water, and ensuring that irrigators only pump adequate water necessary to sustain a standing crop. Inefficient water use indicators include the volume of return flow and runoff, as a percentage of overall pumping capacity. Water consumption by irrigators also depends on the technology adopted. For instance, overhead sprinkler irrigation systems are inefficient compared with drip systems. The regulator should therefore either impose penalties on inefficient technologies or recommend water saving technologies to be adopted by the irrigators.

When all agents face some ecological restrictions on the use of water and complementary resources, sustainability could be achieved without reducing the flow of private and social
benefits to all resource users. Community cooperation in managing natural resources including water and vegetal resources depends crucially on the nature of commitments expected from participating agents. An agent's commitment in turn depends on the benefits from resource using activities the agent engages in.

References


APPENDIX 1

Static optimization in laissez-faire

Commercial farmers

Unconstrained optimization of the profit function (equation 1 in the text)

\[ \eta_t(L,K,S,W) = P(q)q - C_t(L,K,S,W) \]

The first order necessary conditions:

\[ \frac{\partial \eta}{\partial L} = P(t) \frac{\partial q}{\partial L} - C_t'(L) = 0 \quad \ldots (A1.1) \]

\[ \frac{\partial \eta}{\partial K} = P(t) \frac{\partial q}{\partial K} - C_t'(K) = 0 \quad \ldots (A1.2) \]

\[ \frac{\partial \eta}{\partial S} = P(t) \frac{\partial q}{\partial S} - C_t'(S) = 0 \quad \ldots (A1.3) \]

\[ \frac{\partial \eta}{\partial W} = P(t) \frac{\partial q}{\partial W} - C_t'(W) = 0 \quad \ldots (A1.4) \]

Smallholder farmers/households

\[ dU = 0 = \tau_t (M)^{-1} dI - \tau_t (Z)^{\gamma-1} dT + \tau_t (W)^{\gamma-1} dW \]

Utility maximizing conditions are given by the first order conditions:

\[ \frac{\partial U}{\partial I} = \tau_t (M)^{-1} \quad \ldots (A1.5) \]

\[ \frac{\partial U}{\partial T} = -\tau_t (Z)^{\gamma-1} \quad \ldots (A1.6) \]

\[ \frac{\partial U}{\partial W} = \tau_t (W)^{\gamma-1} \quad \ldots (A1.7) \]
The marginal rates of substitution are:

$$\frac{\partial U}{\partial T} = -\frac{i_2 t_1 (Z)^{\gamma-1}}{t_4 t_1 (M)^{\gamma-1}}$$ ... (A1.8)

$$\frac{\partial U}{\partial W} = -\frac{i_2 t_1 (Z)^{\gamma-1}}{i_4 t_1 (W)^{\gamma-1}}$$ ... (A1.9)

$$\frac{\partial U}{\partial I} = \frac{i_2 t_1 (M)^{\gamma-1}}{t_4 t_1 (W)^{\gamma-1}}$$ ... (A1.10)

**Pastoralists**

The objective function for the pastoralists is

Maximize \( NPB = ph - c(x, v, w) \)

$$\frac{\partial NPB}{\partial x} = p \frac{\partial h}{\partial x} \frac{\partial c}{\partial x} = 0$$ ... (A1.11)

$$\frac{\partial NPB}{\partial v} = p \frac{\partial h}{\partial x} \frac{\partial x}{\partial v} \frac{\partial c}{\partial v} = 0$$ ... (A1.12)

$$\frac{\partial NPB}{\partial w} = p \frac{\partial h}{\partial x} \frac{\partial x}{\partial w} \frac{\partial c}{\partial w} = 0$$ ... (A1.13)
APPENDIX 2

The cooperative model is specified as follows.

\[ L = \sum_{i=0}^{n} \rho^i \left( G V_i + \rho_l \lambda_i \left[ x_i + f(x_i, v_i, w_i) - h_i - x_{i+1} \right] + \rho \theta_i \left[ v_i + g(x_i, v_i, w_i) - v_{i+1} \right] + \rho \psi_{i+1} \left[ w_i + r(x_i, v_i, w_i, z_i) - w_{i+1} \right] \right) \]

Maximum principles are derived as below

\[ \frac{\partial L}{\partial v_i} = \rho^i \left( \gamma \frac{\partial N P B_i}{\partial v_i} + \rho_l \lambda_i \left[ 1 + \frac{\partial f}{\partial v_i} \right] + \rho \theta_i \left[ \frac{\partial g}{\partial v_i} + \frac{\partial \theta_i}{\partial v_i} \right] \right) - \rho^i \theta_i = 0 \quad \ldots \text{(A2.1)} \]

\[ \frac{\partial L}{\partial v_i} = \rho^i \left( \gamma \frac{\partial N P B_i}{\partial v_i} + \rho_l \lambda_i \left[ \frac{\partial f}{\partial v_i} - \frac{\partial h_i}{\partial v_i} \frac{\partial x_i}{\partial v_i} \right] + \rho \theta_i \left[ 1 + \frac{\partial g}{\partial v_i} \right] \right) + \rho \psi_{i+1} \left[ \frac{\partial \psi_i}{\partial v_i} \right] - \rho^i \theta_i = 0 \quad \ldots \text{(A2.2)} \]

\[ \frac{\partial L}{\partial w_i} = \rho^i \left( \gamma \frac{\partial N P B_i}{\partial w_i} + \gamma \frac{\partial U_i}{\partial w_i} + \gamma \frac{\partial N P B_i}{\partial w_i} + \rho_l \lambda_i \left[ \frac{\partial f}{\partial w_i} - \frac{\partial h_i}{\partial w_i} \frac{\partial x_i}{\partial w_i} \right] + \right. \]

\[ \left. \rho \theta_i \left[ \frac{\partial \psi_i}{\partial w_i} \right] + \rho \psi_{i+1} \left[ 1 + \frac{\partial \psi_i}{\partial w_i} \right] \right) - \rho^i \psi_i = 0 \quad \ldots \text{(A2.3)} \]
\[ \Rightarrow \lambda_i = \left\{ \gamma_3 \frac{\partial NPB}{\partial x_i} + \rho \lambda_{ri} \left[ 1 + \frac{\partial f}{\partial x_i} \right] + \rho \theta_{ri} \frac{\partial g}{\partial x_i} + \rho \psi_{ri} \frac{\partial r}{\partial x_i} \right\} \quad \text{(A2.4)} \]

\[
\Rightarrow \theta_i = \left\{ \gamma_3 \frac{\partial NPB}{\partial v_i} + \rho \lambda_{ri} \left[ \frac{\partial f}{\partial t} - \frac{\partial h_i}{\partial x_i} \frac{\partial x_i}{\partial v_i} \right] + \rho \theta_{ri} \left[ 1 + \frac{\partial g}{\partial v_i} \right] + \rho \psi_{ri} \left[ \frac{\partial w_i}{\partial v_i} + \frac{\partial r}{\partial v_i} \right] \right\} \quad \text{(A2.5)}
\]

\[
\Rightarrow \psi_i = \left\{ \gamma_1 \frac{\partial u_i}{\partial w_i} + \gamma_2 \frac{\partial U_i}{\partial w_i} + \gamma_3 \frac{\partial NPB}{\partial w_i} + \rho \lambda_{ri} \left[ \frac{\partial f}{\partial w_i} - \frac{\partial h_i}{\partial x_i} \frac{\partial x_i}{\partial w_i} \right] + \rho \theta_{ri} \left[ \frac{\partial g}{\partial w_i} \right] + \rho \psi_{ri} \left[ 1 + \frac{\partial r}{\partial w_i} \right] \right\} \quad \text{(A2.6)}
\]